XXXVII. *Experimental Researches on the Conductive Powers of various Substances, with the application of the Results to the Problem of Terrestrial Temperature.*

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In presenting this memoir to the Society, I feel myself called upon to acknowledge the liberal aid which I have received from the Committee for the disposal of the Annual Government Grant for the Advancement of Science, in the prosecution of the experimental researches in which I have been engaged for a considerable time. The present paper contains an account of a part only of my experiments, with the application of the results of them to the problem of terrestrial temperature. An account of the remaining experiments will be reserved for a future paper. I am likewise bound to express in the strongest terms my obligations to my friends Mr. Fairbairn and Mr. Joule. Without the aid of the former of these gentlemen I should have been unable even to commence the series of experiments which I have now nearly concluded; and among the many ways in which this assistance has been so promptly rendered, I may mention his having constantly placed at my disposal the invaluable services of one of his principal workmen, William Ward, without whose untiring activity and mechanical resources, I should have utterly despaired of bringing my experiments to any successful issue. The value of Mr. Joule's assistance, especially in the commencement of these experiments, will be understood by those who are acquainted with his sound philosophical knowledge and experimental skill. More specific acknowledgement of his services will be called for in my next communication. I now proceed to the subject of this paper.

§ I. *General Experimental Results respecting the Conductive Power of various Substances, and the circumstances by which it is affected.*

1. That I may be able to state the more clearly the nature of the quantity which measures the conductivity of any proposed substance with reference to heat, and the experimental method I have adopted for the determination of its value, it will be necessary to recall the solution of the problem, under one of its simplest aspects, the object of which is to ascertain the manner in which heat is transmitted by conduction through a body under certain conditions. Conceive the body to be bounded by two parallel plane surfaces of indefinite extent, the distance between them being h. Suppose one of these bounding surfaces (which, for convenience, may be termed the *lower* one) to be kept at a uniform and constant temperature t_1, while the temperature of the free space into which the heat radiates from the *upper* surface, is constant and $=\tau$. Also let ζ

denote the temperature at the distance z from the lower surface, when the temperature has become *steady*. The differential equation for the determination of ζ will then be

$$\frac{d}{dz}\left(k\frac{d\zeta}{dz}\right)=0; \quad \ldots \quad (1.)$$

$$\therefore \quad k\frac{d\zeta}{dz}=\mathrm{C}, \text{ a constant.} \quad \ldots \quad (2.)$$

The expression $-k\frac{d\zeta}{dz}$ measures the quantity of heat which passes through a unit of area parallel to either bounding surface, in a unit of time. We may conceive k to be a function of ζ or z, but it is here considered constant. It is the quantity which is always taken to measure the *conductivity*, or *conductive power*, of the mass through which the heat is transmitted, and which can only be determined for different substances by experiment.

Again, let t_2 be the temperature of the upper bounding surface; then will the quantity of heat which radiates from a unit of area of that surface in a unit of time be

$$p(t_2-\tau),$$

p being constant, and independent of the temperatures t_3 and τ, at least for considerable ranges of those temperatures. It measures the *radiating power* of the upper surface of the mass*. Now since the same quantity of heat must pass through a unit of the upper surface as through any unit of area parallel to that surface in the interior of the mass, equation (2.) will become

$$-k\frac{d\zeta}{dz}=p(t_2-\tau). \quad \ldots \quad (3.)$$

Integrating again

$$\mathrm{C}'-k\zeta=p(t_2-\tau)z;$$

and since $\zeta=t_1$ when $z=0$,

$$k(t_1-\zeta)=p(t_2-\tau)z; \quad \ldots \quad (4.)$$

and since $\zeta=t_2$ when $z=h$,

$$\frac{k}{p}=\frac{t_2-\tau}{t_1-t_2}h. \quad \ldots \quad (5.)$$

If h be known and the temperatures t_1, t_2 and τ be observed, this equation will determine $\frac{k}{p}$, the ratio of the conductivity of the substance to the radiating power of its surface, which is very different for different substances. If, however, the upper surface be covered by a thin stratum of any other matter which will assume the temperature of the upper surface of the transmitting mass, p will then be the radiating power of this super-

* I have adopted the approximate law of radiation, as much more simple and convenient than the more exact law of DULONG and PETIT, and sufficiently accurate for my purpose. All the experiments described in this paper aim only at *comparative* results, and most of them have been made under nearly the same thermal conditions. Those requiring any considerable accuracy have been made in the form of *differential* experiments. Hence the use of the approximate law of radiation can lead to no error of any importance in the experimental results.

imposed matter. I made use of mercury for this purpose in my own experiments; and thus, if c denote the radiating power of mercury, we have

$$\frac{k}{c}=\frac{t_2-\tau}{t_1-t_2}h; \quad \ldots \quad (6.)$$

and c being the same, whatever may be the substance experimented on, this formula enables us to compare the conductive powers of different substances, or to determine the absolute numerical values of those powers, when c the radiating power of mercury is known. The determination of these comparative values of k, and not those of $\frac{k}{p}$ (which have more usually been determined), has formed the object of these experimental investigations.

My experiments have been made on a great variety of mineral substances, and on some others also: I shall reserve certain details respecting them for a subsequent part of this paper, giving here the general results at which I have arrived. It should be observed, that k being a linear quantity, like h $\left(\text{as appears from the expression for } \frac{k}{c}\right)$, its numerical value will depend on the unit of length. This unit has been assumed to be *one foot**.

2. Experiments were made for the purpose of ascertaining the conductive powers of calcareous, argillaceous, and siliceous masses in a state of dry powder. The first was obtained from a piece of pure chalk rock, the second from a piece of clay which appeared to have very little admixture of other elements, and the third was obtained from a piece of New Red Sandstone. All were thoroughly dried. The results were as follows:—

	Values of $\frac{k}{c}$.
Calcareous powder	·056
Argillaceous powder	·07
Siliceous powder	·15
A mixture of the two last in equal quantities	·11

3. The following results were obtained for different kinds of rocks:—

Calcareous rocks.

Chalk, as it exists in the general mass of chalk, but well dried	·17
Clunch, from the lower portion of the chalk formation (very moist)	·30
Oolites from three different beds in the Ancaster quarries	·38 ·37 ·37
Statuary marble	·53
Very hard blue mountain limestone from Derbyshire	·55

* The temperatures are expressed in degrees of FAHRENHEIT's thermometer.

Argillaceous substances.	Values of $\frac{k}{c}$.
Dry clay	·27
Very dry	·23
Moist	·37
Siliceous rocks.	
New Red Sandstone (dry)	·25
New Red Sandstone (saturated with water)	·60
Sandstone for building (freestone)	·33
Sandstone for building	·43
Millstone-grit (partially decomposed)	·376
Millstone-grit	·58
Millstone-grit from the coal-shaft at Duckenfield, at depth of 120 feet	·51
Millstone-grit from deeper beds	·65
Millstone-grit from the depth of 1300 feet, and very hard	·726
Millstone-grit from Chapel-le-Frith, used for paving-stone at Manchester, very hard	·75
Millstone-grit from Chapel-le-Frith, used for paving-stone at Manchester, very hard	·76
Old Sedimentary rocks.	
Blue, hard, close-grained rock from Penmaenmaur, used for paving-stone	·5
A similar specimen	·6
Blue, hard, compact slate from Charnwood Forest	·61
Igneous rocks.	
Granite	·53
Scotch granite, used for paving at Manchester	·55
A hard compact rock from North Wales (called *Welsh granite*)	·60
Scotch granite, rather large-grained	·75
Basalt from near Loch Katrine	·53
Syenite from Charnwood Forest	·85
A very hard, close-grained rock from Charnwood Forest	·99
Igneous rock from Loch Katrine	1·0
Basalt from the same locality	·59
Mountsorrel granite	·8

A great number of experiments were also made to determine the influence of pressure, discontinuity, temperature and moisture on the conductive powers of various substances. I proceed to state the general results of them.

4. *Influence of Pressure.*

(1) A block of spermaceti, solidified under a pressure of 850 lbs. per square inch, gave

$$\frac{k}{c}=\cdot 086.$$

Another block, solidified under a pressure of 7500 lbs. per square inch, gave so nearly the same value as only to admit the conclusion that pressure had no sensible influence on the conductive power of this substance.

Instead of taking blocks solidified under the above pressures, I took another compressed, *after solidification*, with a weight of 7500 lbs. per square inch. The result was identical with that obtained from the block solidified under the same pressure.

(2) Wax.—This substance, when uncompressed, gave

$$\frac{k}{c}=\cdot 072,$$

and when compressed by a pressure of 7500 lbs. per square inch,

$$\frac{k}{c}=\cdot 079.$$

The increase is too small to exceed the probable limits of error.

(3) Clay.—The results in this case were as follows:—

Uncompressed clay	$\frac{k}{c}=\cdot 26$
Compressed with a weight of 4300 lbs. per square inch	$\frac{k}{c}=\cdot 30$
Compressed with a weight of 7500 lbs. per square inch . . .	$\frac{k}{c}=\cdot 33$

Here we have a considerable increase due to pressure.

(4) Chalk.—No appreciable increase of conductivity was observed in this case with pressures equal to those above cited.

(5) Mixture of sand and clay:—

Compressed with a weight of 4300 lbs. per square inch . . .	$\frac{k}{c}=\cdot 36$
Compressed with a weight of 7500 lbs. per square inch . . .	$\frac{k}{c}=\cdot 378$

5. *Influence of Temperature.*

The experiments for determining the influence of temperature were confined to two substances which melt at low temperatures, bees'-wax and spermaceti. Denoting the temperatures of the lower and upper surfaces of the blocks respectively by t_1 and t_2, I obtained the following results:—

$$\left.\begin{array}{l} t_1=105^\circ\cdot 3 \\ t_2=\ 85^\circ\cdot 6 \end{array}\right\}, \ \frac{k}{c}=\cdot 088$$

$$\left.\begin{array}{l} t_1=115^\circ\cdot 5 \\ t_2=\ \ 91^\circ\cdot 5 \end{array}\right\}, \frac{k}{c}=\cdot 09$$

$$\left.\begin{array}{l} t_1=126^\circ\cdot 5 \\ t_2=\ \ 95^\circ\cdot 0 \end{array}\right\}, \frac{k}{c}=\cdot 077$$

$$\left.\begin{array}{l} t_1=139^\circ \\ t_2=100^\circ \end{array}\right\}, \frac{k}{c}=\cdot 074.$$

The temperature of complete melting of the wax was about 140°, so that in the last experiments the lower part of the block was very nearly in a melting state, and the molecular condition of a considerable portion of it was doubtless affected. This change is indicated by a decrease of conductivity. Exactly similar results were obtained for spermaceti. So long as the temperature was insufficient to produce any apparent effect on the constitution of these substances, my experiments did not detect any sensible effect on their conductivity. BIOT remarked the diminution of conductivity of a bar of fusible alloy when one extremity was maintained very nearly at its fusing temperature.

With respect to mineral substances of very high temperatures of fusion, there can be little doubt, I conceive, that their conductivity remains unaffected by any temperature, for instance, not exceeding that of boiling mercury. Some experiments were made, partly with the view of corroborating this conclusion, but more especially for testing, in the case of mercury, the approximate truth of the law which asserts that the intensity of radiation from a given surface is proportional to the excess of the temperature of that surface above that of the surrounding space. The following results were obtained by experimenting at different temperatures with the same block of sandstone; t_1 and t_2 denote the same temperatures as above, and τ the temperature of the surrounding atmosphere:—

$$(1)\ \left\{\begin{array}{l} t_1=144^\circ\cdot 5 \\ t_2=123^\circ\cdot 2 \end{array}\right\}, \ \tau=66^\circ, \quad \frac{k}{c}=\cdot 45.$$

$$(2)\ \left\{\begin{array}{l} t_1=241^\circ\cdot 5 \\ t_2=193^\circ\cdot 0 \end{array}\right\}, \ \tau=69^\circ, \quad \frac{k}{c}=\cdot 435.$$

$$(3)\ \left\{\begin{array}{l} t_1=334^\circ\cdot 0 \\ t_2=260^\circ\cdot 5 \end{array}\right\}, \ \tau=78^\circ, \quad \frac{k}{c}=\cdot 425.$$

$$(4)\ \left\{\begin{array}{l} t_1=447^\circ\cdot 8 \\ t_2=326^\circ\cdot 0 \end{array}\right\}, \ \tau=81^\circ, \quad \frac{k}{c}=\cdot 34.$$

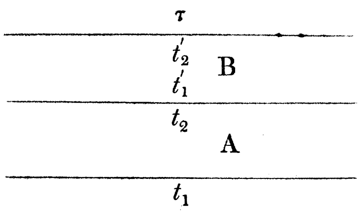
$$(5)\ \left\{\begin{array}{l} t_1=522^\circ\cdot 0 \\ t_2=371^\circ\cdot 0 \end{array}\right\}, \ \tau=85^\circ\cdot 5, \ \frac{k}{c}=\cdot 33.$$

Hence if we assume k to be constant for these different temperatures, c the radiating power of mercury must increase with the increase of the difference $t_2-\tau$. The increase however is not considerable while $t_2-\tau$ varies from 50° to 120° or 130°, as in the two first of the above experiments. Within those limits the law of radiation, as usually assumed, will be approximately true. It was desirable to corroborate this conclusion with reference to mercury, on account of its being the substance from which the radiation took place in all my experiments. The increase of radiation indicated by the above experiments is in general accordance with the law of DULONG and PETIT, but it is smaller in amount.

6. *Effect of Discontinuity.*

In all the preceding cases the substances experimented on were in unbroken continuous small masses. It is important to inquire how far their conductivity is affected by a breach of this perfect continuity. Several experiments were made for this purpose; but before I proceed to state these experimental results, it may be better (as in art. 1) to deduce the theoretical results with which they must be compared for the purpose of determining the additional coefficient, or constant, which must be introduced on the hypothesis that the discontinuity in the conducting substance produces a discontinuity in the law according to which the temperature decreases as the heat passes from one bounding surface to the other. These surfaces being supposed parallel and of indefinite extent, as before, for a mass of one substance (A), conceive another mass (B) of different conductivity, and bounded also by parallel plane surfaces, to be placed upon the former. Let t_1 denote the constant temperature at which the lower surface of A (fig. 1) is maintained; t_2 that of its upper surface; t'_1 the temperature of the lower surface of B in contact with the upper surface of A; t'_2 that of the upper surface of B; and τ that of surrounding space. I shall assume that the quantity of heat which flows through a unit of area of the surfaces of junction, in a unit of time,

Fig. 1.



$$=q(t_2-t'_1),$$

where q is independent of t_2 and t'_1. Then, since the same quantity of heat when the temperature is steady, must pass through each parallel surface, we must have

$$q(t_2-t'_1)=p(t'_2-\tau), \quad \ldots \ldots \ldots \quad (1.)$$

p being the radiating power of the upper surface of B. We have also (art. 1.)

$$-k_1\frac{d\zeta}{dz}=p(t'_2-\tau),$$

and therefore

$$C-k_1\,\zeta=p(t'_2-\tau)z,$$

$$C-k_1t_1=0,$$

$$\therefore \quad k_1(t_1-\zeta)=p(t'_2-\tau)z,$$

and

$$k_1(t_1-t_2)=p(t'_2-\tau)h_1, \quad \ldots \ldots \ldots \quad (2.)$$

where h_1=thickness of A.

In like manner we obtain

$$k_2(t'_1-t'_2)=p(t'_2-\tau)h_2, \quad \ldots \ldots \ldots \quad (3.)$$

where h_2=thickness of B.

Equations (1.), (2.) and (3.) will determine t'_2, t_1 and t'_2, in terms of t_1 and τ, and the other quantities involved. They may be written

$$\left.\begin{aligned} t_1-t_2&=\frac{p}{k_1}(t'_2-\tau)h_1, \\ t_2-t'_1&=\frac{p}{q}(t'_2-\tau), \\ t'_1-t'_2&=\frac{p}{k_2}(t'_2-\tau)h_2. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (\alpha.)$$

Adding them together, we have

$$t_1-t'_2=\left(\frac{ph_1}{k_1}+\frac{ph_2}{k_2}+\frac{p}{q}\right)(t'_2-\tau),$$

or writing $t_1-\tau-(t'_2-\tau)$ for $t_1-t'_2$,

$$t_1-\tau=\left(\frac{ph_1}{k_1}+\frac{ph_2}{k_2}+\frac{p}{q}+1\right)(t'_2-\tau);$$

$$\frac{ph_1}{k_1}+\frac{ph_2}{k_2}+\frac{p}{q}=\frac{t_1-\tau}{t'_2-\tau}-1$$

$$=\frac{t_1-t'_2}{t'_2-\tau},$$

$$\frac{p}{q}=\frac{t_1-t'_2}{t'_2-\tau}-p\left(\frac{h_1}{k_1}+\frac{h_2}{k_2}\right),$$

or if $h_1+h_2=h$, and $k_1=k_2=k$,

$$\frac{p}{q}=\frac{t_1-t'_2}{t'_2-\tau}-\frac{ph}{k}. \quad \ldots\ldots\ldots\ldots \quad (4.)$$

If (t_1), (t'_2) and (τ) denote the values which t_1, t'_2 and τ would have in an experiment with an *undivided* block of equal length and of the same substance, we shall have

$$\frac{ph}{k}=\frac{(t_1)-(t_2)}{(t_2)-\tau} \quad \big(\text{art. 1, equation (5.)}\big),$$

and therefore

$$\frac{p}{q}=\frac{t_1-t'_2}{t'_2-\tau}-\frac{(t_1)-(t_2)}{(t_2)-(\tau)}\ldots\ldots\ldots\ldots\ldots \quad (5.)$$

7. To apply this formula to the experimental determination of $\frac{p}{q}$, I took two equal blocks of the same piece of sandstone. One of these was split transversely, and the experiments conducted as hereafter described (see page 845). The undivided block gave

$$\frac{(t_1)-(t_2)}{(t_2)-(\tau)}=\cdot 315.$$

(1) In the first experiment with the divided block, one portion of it was placed in close contact with the other, without cement of any kind between them. This gave

$$\frac{t_1-t'_2}{t'_2-\tau}=\cdot 44,$$

and therefore
$$\frac{p}{q}=\cdot 125.$$

(2) In the next experiment, the two portions of the divided block were cemented together with plaster of Paris, applied moist and left to dry and harden, and forming a layer of about $\frac{1}{20}$th of an inch in thickness. I thus obtained
$$\frac{t_1-t'_2}{t'_2-\tau}=\cdot 365,$$
and
$$\frac{p}{q}=\cdot 05.$$

(3) Fine dry powdered clay was then put between the two portions of the divided block. The result was
$$\frac{t_1-t'_2}{t'_2-\tau}=\cdot 415,$$
and
$$\frac{p}{q}=\cdot 1.$$

(4) Moist clay was then used to cement the two portions of the block, which were pressed together for twelve hours. In this case we had
$$\frac{t_1-t'_2}{t'_2-\tau}=\cdot 384,$$
and
$$\frac{p}{q}=\cdot 069.$$

These experiments were made with a temperature t_1 nearly equal to that of boiling water. To ascertain whether q was independent of the temperature, experiments (3) and (4) were repeated at lower temperatures, t_1 being about 150°. The first gave
$$\frac{p}{q}=\cdot 085,$$
and the second,
$$\frac{p}{q}=\cdot 058.$$

These values are respectively about one-sixth less than those obtained in experiments (3) and (4). They may be considered as corroborating the approximate truth of the assumed law that the coefficient q is independent of the temperature, at least when the difference of temperature is not too great.

Hence, then, the values of $\frac{q}{p}$, in the four cases above stated, are

$$(1)\quad \frac{q_1}{p}=8,$$

$$(2)\quad \frac{q_2}{p}=20,$$

$$(3)\quad \frac{q_3}{p}=10,$$

$$(4)\quad \frac{q_4}{p}=14.$$

The cases (2) and (4) are those in which the junction was doubtless rendered most complete, by means of plaster of Paris in the first case (applied in a moist state and allowed to dry), and in the latter case, by means of moistened clay, applied under considerable pressure. These cases, it will be observed, correspond to the largest values of q, and, therefore, to the most rapid transmission of heat through the surface of junction for any given difference of temperature on each side of it.

8. The influence of this kind of discontinuity in the conducting mass is manifestly of great importance in the conduction of terrestrial heat from the interior of the earth to its surface through a series of different strata. If we have several superimposed strata, each having the same conductivity, but separated by planes of discontinuity, as in the preceding experiments, it may be convenient to know what must be the conductive power of a *continuous* mass of equal thickness, in order that the temperatures of the upper surfaces of the continuous and discontinuous masses may be the same, the temperatures of their lower surfaces being equal, as well as the radiating powers of their upper surfaces. The quantities of heat transmitted in the two cases will then be the same.

Let us suppose the conducting mass to have the same conductivity throughout, but to be divided into $n+1$ portions by n planes of discontinuity, h_1, h_2, ... h_n being their respective thicknesses. Let us also suppose the coefficient q to be the same for each discontinuity. Instead of the equations (α) (art. 6), we shall have the following system:—

$$\left.\begin{aligned}
t_1 - t_2 &= \frac{p}{k}(t_2^{(n)}-\tau)h, \\
t_2 - t_1' &= \frac{p}{q}(t_2^{(n)}-\tau), \\
t_1' - t_2' &= \frac{p}{k}(t_2^{(n)}-\tau)h_1, \\
t_2' - t_1'' &= \frac{p}{q}(t_2^{(n)}-\tau), \\
\&c. &= \&c. \\
t_1^{(n-1)} - t_2^{(n-1)} &= \frac{p}{k}(t_2^{(n)}-\tau)h_{n-1}, \\
t_2^{(n-1)} - t_1^{(n)} &= \frac{p}{q}(t_2^{(n)}-\tau), \\
t_2^{(n)} - t_2^{(n)} &= \frac{p}{k}(t_2^{(n)}-\tau)h_n;
\end{aligned}\right\} \quad \ldots \ldots \quad (\beta)$$

and adding,

$$t_1-t_2^{(n)}=\frac{p}{k}(t_2^{(n)}-\tau)(h+h_1+h_2+\ldots+h_n)+n\frac{p}{q}(t_2^{(n)}-\tau),$$

$$t_1-t_2^{(n)}=\frac{p}{k}\left(h+h_1+h_2+\ldots+n\frac{p}{q}\right)(t_2^{(n)}-\tau),$$

$$=\left(\frac{p}{k}\text{H}+n\frac{p}{q}\right)(t_2^{(n)}-\tau),$$

when $H=h+h_1+h_2+\ldots+h_n$; and writing $t_1-\tau-(t_2^{(n)}-\tau)$ for $t_1-t_2^{(n)}$, we have

$$\frac{p}{k}H+n\frac{p}{q}=\frac{t_1-\tau}{t_2^{(n)}-\tau}-1$$

$$=\frac{t_1-t_2^{(n)}}{t_2^{(n)}-\tau}.$$

Let k' be the required conductivity of a mass whose thickness is H, and of which the extreme temperatures are t_1 and $t_2^{(n)}$ as in the divided mass. Then, since the radiating power (p) is also the same,

$$\frac{t_1-t_2^{(n)}}{t_2^{(n)}-\tau}=\frac{p}{k'}\cdot H, \text{ (art. 1)},$$

and therefore

$$\frac{p}{k}H+n\frac{p}{q}=\frac{p}{k'}\cdot H,$$

$$\therefore \quad p\left(\frac{1}{k'}-\frac{1}{k}\right)=n\frac{p}{q}\cdot\frac{1}{H},$$

$$\frac{k-k'}{k'}=n\frac{p}{q}\cdot\frac{k}{p}\cdot\frac{1}{H},$$

and therefore,

$$\frac{k'}{k}=\frac{1}{1+n\frac{p}{q}\cdot\frac{k}{pH}} \quad \ldots\ldots\ldots\ldots \quad (6.)$$

As an example of this formula, let us take $\frac{p}{q}=\frac{1}{10}$ as determined in case (3.) of art. 7; and $\frac{k}{p}=\cdot52$ (its value for the sandstone used in the experiments), the unit of length being one foot. Then

$$\frac{k'}{k}=\frac{1}{1+\frac{n}{20H}}.$$

Suppose H=100 feet, and that there are 100 discontinuities,

$$\frac{k'}{k}=\frac{1}{1+\frac{1}{20}}=1-\frac{1}{20} \text{ nearly;}$$

so that with a discontinuity on the average for every foot, the effect would only be equivalent to a diminution of $\frac{1}{20}$th of the conductive power; and a discontinuity every 6 inches would be equivalent to a diminution of about $\frac{1}{10}$th of that power, in the particular case now selected for illustration.

9. In the practical application of these researches to the case of the earth's crust, (equation 6.) (art. 8) is so important that it may be worth while to verify it by a somewhat different process. It appears by equation (4.) that the temperature t_2' is entirely independent of h_1 or h_2, the distances of the plane of discontinuity from the terminal planes, since the equation only involves the sum of h_1 and h_2, or h. We may therefore conceive the plane of discontinuity indefinitely near to the terminal plane of which the temperature is t_1. In like manner we may suppose all the planes of discontinuity, in the

general case, placed indefinitely near to the same terminal plane. But we shall then have, in the limit,

$$\begin{aligned} t_2 &= t_1 \\ t_2' &= t_1' \\ &\text{\&c.} \\ t_2^{(n)} &= t_1^{(n)}; \end{aligned}$$

and consequently the alternate equations of the system of equations (β.) (art. 8) will become

$$\begin{aligned} t_1 - t_1' &= \frac{p}{q}(t_2^{(n)} - \tau), \\ t_1' - t_1'' &= \frac{p}{q}(t_2^{(n)} - \tau), \\ \text{\&c.} &= \text{\&c.} \\ t_1^{(n-1)} - t_1^{(n)} &= \frac{p}{q}(t_2^{(n)} - \tau). \end{aligned}$$

Adding, we have
$$t_1 - t_1^{(n)} = n\frac{p}{q}(t_2^{(n)} - \tau),$$

or
$$t_1 - t_2^{(n)} - (t_1^{(n)} - t_2^{(n)}) = n\frac{p}{q}(t_2^{(n)} - \tau);$$

$$\therefore \quad \frac{t_1 - t_2^{(n)}}{t_2^{(n)} - \tau} = \frac{t_1^{(n)} - t_2^{(n)}}{t_2^{(n)} - \tau} = n\frac{p}{q}.$$

The conductivity of the actual mass being k, and the thickness H, we have

$$\frac{p\text{H}}{k} = \frac{t_1^{(n)} - t_2^{(n)}}{t_2^{(n)} - \tau} \text{ (art. 1)};$$

and k' being the conductivity of a mass of the same thickness, and of which the terminal temperatures are, by hypothesis, t_1 and $t_2^{(n)}$, we have

$$\frac{p\text{H}}{k'} = \frac{t_1 - t_2^{(n)}}{t_2^{(n)} - \tau}.$$

Hence
$$p\left(\frac{1}{k'} - \frac{1}{k}\right) = n\frac{p}{q} \cdot \frac{1}{\text{H}},$$

and
$$\frac{k'}{k} = \frac{1}{1 + n\frac{p}{q} \cdot \frac{k}{p\text{H}}},$$

as before.

10. *Effect of Moisture.*

The following experiments were made to ascertain the influence of *moisture* on the conductive powers of rocks.

(1) *Calcareous Rocks.*—I took a block of chalk from the lower part of the Chalk formation near Cambridge. It is provincially called *clunch*, and is frequently used as building-stone, where it can be well defended against the disintegrating influences of the

atmosphere. It is more compact than ordinary chalk. The specimen used in the experiment was very moist. It gave

$$\frac{k}{c}=\cdot 3;$$

and when thoroughly saturated with moisture, it gave again

$$\frac{k}{c}=\cdot 3.$$

The same block, when thoroughly dried, gave

$$\frac{k}{c}=\cdot 19.$$

In the first case, the block weighed

6808·5 grains

immediately after the experiment. In the second case, when perfectly saturated, the weight was 7602 grains before the experiment, and 7503 grains after it, while, when thoroughly dried, it was

6187·5 grains.

Hence in the first case the block may be considered to have contained 621 grs. of water, nearly $\frac{1}{10}$th of the weight of the dry chalk; and in the second, 1315·5 grs., while the conductive power in the two cases was the same. It should be observed, however, that the temperature of the lower surface of the block in the first experiment was 211°·5, and that of the upper surface about 163°; while in the latter the two temperatures were only 123°·5 and 101°·4 respectively. It is probable that the higher temperature might increase the conductivity by the more rapid conversion of the water into vapour.

These results were verified by almost identical results obtained from another similar block of chalk.

The specimen of *clunch* on which I experimented did not, as I conceived, afford any certain test of the quantity of moisture ordinarily contained in the Chalk as it exists in the general mass of the chalk formation. To obtain a better test, I afterwards procured a specimen from the same place near Cambridge as that from which the above-mentioned specimens were obtained. It was taken from a spot which had not been immediately exposed to the atmosphere, but it contained a quantity of moisture sufficient to render its colour much darker than the white of dry chalk. Soon after it was taken from the mass it weighed

4966·4 grains,

and after being thoroughly dried by a fire for several days, it weighed

4154·1 grains.

It therefore lost 812·3 grs., or nearly one-fifth of its weight when dry. This proportion is about the same as in the case above mentioned, in which the chalk was *saturated* with water. The preceding value (·3) of $\frac{k}{c}$ may therefore be considered as applicable to the general mass of the Lower Chalk near Cambridge. This, I have no doubt, is near the maximum value for chalk, the general structure of this *clunch* of the Lower Chalk being much more compact, and approaching nearer to the character of *rock* than the mass of the Upper Chalk. I should estimate the value of $\frac{k}{c}$ for chalk in the general mass at about ·25.

Similar experiments were made on an Oolitic block from the quarries at Ancaster. It gave, when saturated with moisture,

$$\frac{k}{c}=\cdot 34,$$

and when dry,

$$\frac{k}{c}=\cdot 30.$$

In the first case the weight was

8633 grains

immediately after the experiment, and

8041·5 grains

when dry. The quantity of water contained in it, when saturated, may therefore be estimated at

591·5 grains,

so that the effect on the conductivity in this instance was much less in proportion to the quantity of moisture, than in chalk. The result was verified by similar experiments on another block. Both blocks had given $\frac{k}{c}=\cdot 37$ when first tried. They had probably imbibed considerable moisture, the season being at the time very wet. These experiments were made at about the lower temperatures above mentioned.

(2) *Arenaceous Rocks.*—A block of New Red Sandstone was very much affected in its conductivity by being saturated with moisture. When saturated it gave

$$\frac{k}{c}=\cdot 60,$$

and when dry,

$$\frac{k}{c}=\cdot 25.$$

In the first case the weight, after the experiment, was

7442 grains,

and when dry, its weight was

6883 grains;

and therefore, the weight of the contained water was

559 grains.

The effect on the conductivity was much greater than in any other case. The quantity of water imbibed in the saturated state was about 350 grains greater than the last-stated amount. These 350 grains were lost during the experiment, though made at temperatures little exceeding 100°.

Another block of harder sandstone than the preceding gave, when dry,

$$\frac{k}{c}=\cdot 49,$$

when its weight was

9221·5 grains.

When moist, it gave

$$\frac{k}{c}=\cdot 65,$$

its weight being then

9434 grains

after the experiment. And again, when saturated with moisture, it gave

$$\frac{k}{c}=\cdot 62,$$

its weight being

9547 grains

after the experiment. The temperature of the lower surface of the block in each of these experiments was 211°·0.

We have the same conclusion here as with the chalk—that an increase of conductivity is produced by an increase of moisture, but that the conductivity ceases to increase considerably before we arrive at the state of saturation, when it would appear to be somewhat less than its maximum value. We have also the same conclusion from a similar experiment on a block of sandstone from the Millstone Grit, and made at like temperatures with the preceding one. It gave, when dry,

$$\frac{k}{c}=\cdot 55,$$

and then weighed

8999·5 grains.

When moist,

$$\frac{k}{c}=\cdot 65,$$

and it weighed

9210 grains.

When saturated,

$$\frac{k}{c}=\cdot 61,$$

its weight being

9291 grains.

The quantity of water absorbed in both these last cases was much less than in the Chalk and New Red Sandstone, as well as the effect on the conductivity.

(3) *Argillaceous Rocks.*—A block of very dry clay gave

$$\frac{k}{c}=\cdot 23,$$

and a moist block gave

$$\frac{k}{c}=\cdot 37.$$

The weight of the moist block *before* the experiment was

6088·5 grains.

Its weight immediately after the experiment was accidentally not observed, but twelve hours afterwards it was

5687·5 grains.

It probably lost about 150 grains during the experiment, which would make the weight taken as in the preceding cases equal to about

5840 grains.

Its weight after being thoroughly dried was

5251 grains.

Consequently, the absorption of a quantity of water =589 grains, which was nearly one-tenth of the weight of the block, increased the conductive power from ·23 to ·37.

(4) *Very hard Rocks.*—A block of Millstone Grit gave the following results:—when dry,

$$\frac{k}{c}=\cdot 71,$$

weight = 9925 grains.

After being immersed in water about two days,

$$\frac{k}{c}=\cdot 69,$$

weight = 9956 grains.

From a block of close Palæozoic rock I obtained, when dry,

$$\frac{k}{c}=\cdot 52,$$

weight = 11085 grains;

and after immersion in water the same time as the last block,

$$\frac{k}{c}=\cdot 53,$$

weight = 11098 grains.

In these cases the absorbing power was very small, and the difference of the conductive powers indicated by the experiments scarcely exceeds the limits of error.

General Summary of the preceding Results.

11. The conductive powers of calcareous, argillaceous, and siliceous substances—those which compose the great mass of the earth's crust—are in the order in which I have now named these substances, when masses of them are formed by the simple aggregations of each substance previously reduced to dry powder, the first being the worst conductor. The conductive power of Chalk, as it exists in mass, may be estimated

as varying from 2·5 to 3·0; and very hard and compact Mountain Limestone may be estimated at 5·5. These, according to my results, may be taken as the approximate limits for any rocks containing a sufficient quantity of lime to be designated as calcareous rocks.

Arenaceous and siliceous rocks have a somewhat wider range of conductivity. That of friable New Red Sandstone was about ·25, while that of very hard Millstone Grit rises to as much as ·75. With respect to argillaceous rocks, my experiments have been less complete. They show, however, that when not indurated, and in that state of dryness or moisture in which they may be supposed generally to exist in mass, their conductive power may be taken at about ·3. I have not tried any hard clay rock, but, judging from other cases, I should expect it to rise to about ·6.

The igneous rocks have uniformly high conductive powers. In ten cases they varied from ·53 to 1, the mean of them being ·72. In the old sedimentary rocks (Cambrian or Silurian), not calcareous, they appear to vary from about ·5 to ·6, or probably still higher.

It is manifest that whatever may be the mineral constituents of a rock, its conductive power depends very much on its degree of hardness and compactness, that kind of induration which results from some chemical action; while that compactness and comparative hardness which arise merely from mechanical pressure, have less influence than might have been anticipated. In such substances as wax and spermaceti, it is not appreciable in my experiments; nor does it appear to be so in pure calcareous masses like chalk. In clay, however, the effect is very sensible. Dry sand is not sufficiently adhesive to be formed into blocks convenient for the purpose of experiment, even under very heavy pressure. Pressure would probably have little effect on the conductivity.

Temperature has a considerable effect on the conductivity, when it approaches the temperature of fusion. BIOT showed this to be the case with certain alloys which are easily fusible; and I have also found the same true for wax and spermaceti. There seems no reason, however, to suppose that temperatures differing sufficiently from those of fusion, produce much effect on the conductivity.

The conduction of heat is very sensibly affected by *discontinuity* in the conducting mass. I have shown, however, that in the case of a sandstone of about a mean conducting power, a division of the mass into beds of one foot in average thickness would not diminish the conductive power by more than about one-twentieth part.

In rocks which are great absorbents of water, the conductivity is much affected by the degree of moisture. The effect increases with the quantity of moisture up to a certain point, which, however, appears to be considerably short of *saturation*. It would appear slightly to decrease again. In chalk, friable sandstone, and clay, the conductive power is increased in a considerable ratio. Hard sandstone, and any highly indurated rocks, are comparatively bad absorbents, and their conductivity appears to be little affected by any moisture they are capable of imbibing.

§ II. *Comparison of Theoretical Deductions from the preceding Experimental Results, with Observations on Terrestrial Temperature at various depths.*

12. A considerable number of observations have been made (as is well known) to ascertain the temperature of the earth at considerable depths beneath its surface, and the law according to which that temperature increases in descending. The existence of increased heat at considerable depths has long been established beyond all doubt, and the law of increase in a considerable number of localities may be considered as approximately and somewhat roughly determined to be—that the increase of temperature above that of the mean temperature at the surface in any proposed locality, is proportional to the depth beneath the surface.

Now I shall not attempt to discuss the numerous observations by which this approximate law has been established, but shall merely cite a few instances in which the observations may be fully relied on, and in which also the great depth to which they were extended leaves less liability to serious error than in many other cases.

The Puits de Grenelle at Paris is an Artesian well, which extends to the depth of 546 metres, through strata which must all have low conductive powers. It first penetrates through upwards of 41·5 metres of the older tertiaries; afterwards through the chalk to the depth of nearly 500 metres, and finally through argillaceous beds for the remaining depth. The whole process and its results are very fully described in the sixth volume of ARAGO's Works (p. 399). Commencing at the depth of the caves of the Observatory at Paris, where the temperature is constant, and equal to 11°·7 C., the increase of temperature for the whole depth was at the rate of 1° FAHR. for 60 feet. The following Table will also explain the extent of the deviation from uniformity in the progressive increase of temperature in descending (p. 388). It gives for different depths the increase of depth corresponding to an increase of 1° C.

Depths. metres. metres.	Increase of Depth for 1° C. metres.
From 28 to 66	31·1
From 66 to 173	30·6
From 173 to 248	20·8
From 248 to 298	22·8
From 298 to 400	62·5
From 400 to 505	38·9
From 505 to 548	33·0

This Table exhibits a remarkable anomaly in the rate of increase in the central beds of the chalk, for which there appears to be no adequate reason in the nature of the beds themselves as described by ARAGO (p. 426).

ARAGO also gives an account, sent to him by HUMBOLDT, of an Artesian well at Neu-Saltzwerk in Westphalia. It had reached the depth of 644·5 metres, nearly 100 metres deeper than the Puits de Grenelle. The observed temperatures give an increase of about 1° FAHR. for 54 feet of depth. Unfortunately, no geological details are given respecting the nature of the strata through which this well has been sunk. It is merely stated, incidentally, that it penetrated the lower beds of the lias. Another bore is also referred to by HUMBOLDT in his description of the above, in which also the temperature appears to have been very carefully observed by MM. DE LA RIVE and MARCET, near Geneva. The depth was 225 metres, and the observed temperature gave an increase of 1° FAHR. for about 55 feet. Here again no mention is made of the beds which were penetrated.

A very deep Artesian well also exists at Mondorff, in the Grand Duchy of Luxembourg. The account of it is given by ARAGO (p. 397). We have in this case the geological formations through which the well has penetrated.

	metres.
Lias	54·11
Keuper	206·02
Muschelkalk	142·17
Grès bigarré and Grès Vosgien	311·46
Old Schistose Rocks	16·24
Whole depth	730·00

The increase of temperature was at the rate of 1° FAHR. for about 57 feet.

In our own country there are two deep shafts of coal-mines which have afforded good opportunities of making observations of this kind. One is at Monkwearmouth near Sunderland. Professor PHILLIPS found an increase there of 1° FAHR. for about 60 feet in depth. The mouth of this shaft must, I presume, be in the Magnesian limestone, which, with the subjacent formations of the depth of about 1700 feet, must have been penetrated by the shaft.

There is also a coal shaft at Duckenfield, near Manchester, which is to be carried to the depth of upwards of 2000 feet. It has already attained the depth of more than 1400 feet, and from the depth of 700 feet downwards, the proprietor, Mr. ASTLEY, has obligingly had a series of careful observations made at my request. The rate of increase of temperature between the depths of 700 feet and 1330, was 1° FAHR. for about 65 feet. The shaft penetrates through a number of beds of sandstone and shale, of which some are extremely hard and compact. They belong to the Millstone Grit and Coal formations.

I have selected the above instances as those in which observations fully entitled to our confidence have been made at great depths, and therefore likely to be, as far as possible, free from the influences of local causes. But I may also add, that a considerable number of observations made in the coal-mines of this country give the rate of increase as equal to about 1° FAHR. for 60 feet of depth; and the same result has been arrived at by M. WALFERDIN respecting the rate of increase in the Chalk throughout a large portion of Northern France. Each of these results must be considered as the mean rate of increase in the particular well or shaft in which it has been obtained. In most cases the exact temperatures at different depths have not been observed at the only time when they can be obtained with accuracy, *i. e.* during the sinking of the shaft or well. It will be remarked, however, that there are great irregularities in the rate of increase in the 'Puits de Grenelle,' and observation appears to indicate similar variations in other localities; but it is easily conceivable that such variations may be due to local and superficial causes, and that they may disappear at sufficient depths beneath the surface. Assuming this to be the case, and neglecting the effects of local causes, the preceding results of observation would lead to the conclusion that the rate of increase of temperature in descending beneath the earth's surface is nearly uniform in each locality, and nearly the same in different localities. At the same time, many observations indicate material deviations from this equality of the rate of increase in different places, and to these I shall again refer; but the general conclusion which has usually been deduced from observations on terrestrial temperature is that above enunciated.

Now if an enormous sphere, like the earth, were originally heated to any degree, and were then left to cool by radiation into surrounding space, for a sufficient length of time; and supposing, moreover, the conductive power of the mass, or at least of its more superficial portion, to be uniform—then the law of temperature above enunciated as deduced from observation, would be the actual law in the case now supposed, at points not remote from the surface of the sphere. This presumed coincidence between the results of theory and observation has naturally led to a very general adoption of the theory which assigns the existing terrestrial temperature entirely to a primitive heat, of which the remains, though producing a comparatively feeble effect within the range of those depths to which we can penetrate, may yet produce an enormous temperature in the more central portions of the globe. The investigations on this subject, however, have hitherto been very imperfect with respect to the determination of the law of increase of temperature in that stratified envelope of the earth which consists of so many layers of substances possessing, as I have now shown, such different conductive powers. I shall endeavour to supply this deficiency; and for this purpose we must first solve the following problem:—

13. If any number of strata, bounded by parallel surfaces of indefinite extent, be superimposed on each other, the conductive power (k) for each stratum, and the transmitting power (q) for each discontinuity, being different, to find the law of temperature in the mass, the lowest surface being maintained at the constant temperature t_1, and the temperature of the atmosphere being τ.

Recurring to the equations (β.) (art. 8), we shall have, giving to k and q their different values, k, k_1, k_2 ..... k_n and q_1, q_2 ..... q_n respectively,

$$\left.\begin{aligned}
t_1-t_2&=\frac{p}{k}(t_2^{(n)}-\tau)h,\\
t_2-t'_1&=\frac{p}{q_1}(t_2^{(n)}-\tau),\\
t'_1-t'_2&=\frac{p}{k_1}(t_2^{(n)}-\tau)h_1,\\
t'_2-t''_1&=\frac{p}{q_2}(t_2^{(n)}-\tau),\\
\&c. &= \&c.\\
t_1^{(n-1)}-t_2^{(n-1)}&=\frac{p}{k_{n-1}}(t_2^{(n)}-\tau)h_{n-1},\\
t_2^{(n-1)}-t_1^{(n)}&=\frac{p}{q_n}(t_2^{(n)}-\tau),\\
t_1^{(n)}-t_2^{(n)}&=\frac{p}{k_n}(t_2^{(n)}-\tau)h_n;
\end{aligned}\right\} \qquad (\gamma.)$$

and adding, we have

$$t_1-t_2^{(n)}=p(t_2^{(n)}-\tau)\left\{\frac{h}{k}+\frac{h_1}{k_1}+\ldots+\frac{h_n}{k_n}+\frac{1}{q_1}+\frac{1}{q_2}+\ldots+\frac{1}{q_n}\right\};$$

when, writing $t_1-\tau-(t_2^{(n)}-\tau)$ for $t_1-t_2^{(n)}$),

$$t_1-\tau=(t_2^{(n)}-\tau)\left\{1+\frac{ph}{k}+\frac{ph_1}{k_1}\ldots+\frac{ph_n}{k_n}+\frac{p}{q_1}+\frac{p}{q_2}\ldots\ldots+\frac{p}{q_n}\right\}$$

$$=(t_2^{(n)}-\tau)\left\{1+\Sigma\left(\frac{ph}{k}\right)+\Sigma\left(\frac{p}{q}\right)\right\},$$

$$\therefore\quad t_2^{(n)}-\tau=\frac{t_1+\tau}{1+\Sigma\left(\frac{ph}{k}\right)+\Sigma\frac{p}{q}}.$$

The value of $t_2^{(n)}-\tau$ being thus determined, the successive equations of $(\gamma.)$ will determine t_2, t'_1, $t'_2\ldots\ldots t_1^{(n)}$.

For the $(r+1)$th stratum we have

$$-\frac{d\zeta}{dz}=\frac{p}{k_r}(t_2^{(n)}-\tau)$$

$$=\frac{p}{k_r}\,\frac{t_1-\tau}{1+\Sigma\left(\frac{ph}{k}\right)+\Sigma\left(\frac{p}{q}\right)}, \qquad (1.)$$

which gives the rate of decrease of temperature in the $(r+1)$th stratum in ascending.

Integrating this equation, we have

$$t_1^{r}-\zeta=\frac{p}{k_r}\cdot\frac{t_1-\tau}{1+\Sigma\left(\frac{ph}{k}\right)+\Sigma\left(\frac{p}{q}\right)}\cdot z,$$

whence, substituting for $t_1^{(r)}$, ζ is determined for any point in the $(r+1)$th stratum.

14. Let us now suppose that we have, in another locality, another system of superimposed strata similar to the above, in which the quantities $p\ k\ h$ &c. are designated by $p'\ k'\ h'$ &c., and for which t_1 and τ are the same as in the former system. Then shall

we have
$$-\frac{d\zeta'}{dz'}=\frac{p'}{k'_{r'}}\cdot\frac{t_1-\tau}{1+\Sigma\left(\frac{p'h'}{k'}\right)+\Sigma\left(\frac{p'}{q'}\right)}; \quad \ldots \ldots \quad (2.)$$

which gives the rate at which the temperature in the $(r'+1)$th stratum decreases in ascending. Comparing this with the rate in the former system, we have

$$\frac{-\frac{d\zeta}{dz}}{-\frac{d\zeta'}{dz'}}=\frac{p}{p'}\cdot\frac{k'_{r'}}{k_r}\cdot\frac{1+\Sigma\frac{p'h'}{k'}+\Sigma\frac{p'}{q'}}{1+\Sigma\frac{ph}{k}+\Sigma\frac{p}{q}}. \quad \ldots \ldots \quad (3.)$$

To form a general estimate of the numerical values of $\Sigma\frac{ph}{k}$ and $\Sigma\frac{p}{q}$, let us suppose the value of $\frac{k}{p}$ for each of the $n+1$ strata to equal ·5, which is nearly a mean value of $\frac{k}{c}$, as above determined, one foot being the unit of length. Then will $\frac{p}{k}=2$, and

$$\Sigma\frac{ph}{k}=\frac{p}{k}\cdot\Sigma h,$$
$$=2\text{H},$$

when H is the aggregate thickness of the system of strata. Again the number of discontinuities in the $n+1$ strata will $=n$; so that if we take a mean value of $\frac{p}{q}$, we shall have
$$\Sigma\frac{p}{q}=n\frac{p}{q};$$
and if we take $\frac{p}{q}=\frac{1}{10}$, which is greater than the mean of the values given above (art. 7), we have
$$\Sigma\frac{p}{q}=\frac{n}{10}.$$

Hence if twice the aggregate thickness of the strata, expressed in feet, be much greater than one-tenth of the number of strata, and also much greater than unity, the denominator of the expression in equation (3.) will reduce itself nearly to $\Sigma\frac{ph}{k}$; and in like manner, under similar conditions, the numerator will be reduced nearly to $\Sigma\frac{p'h'}{k'}$. Consequently, we shall thus have

$$\frac{-\frac{d\zeta}{dz}}{-\frac{d\zeta'}{dz'}}=\frac{p}{p'}\cdot\frac{k'_{r'}}{k_r}\cdot\frac{\Sigma\frac{p'h'}{k'}}{\Sigma\frac{ph}{k}} \text{ nearly.} \quad \ldots \ldots \quad (4.)$$

15. There are two particular cases which it is important to notice: first, that in which the strata particularized by the conductive powers k_r and $k'_{r'}$, are each of *large thickness* compared with the aggregate thickness of the remaining strata; and, secondly, that in which these two strata are of very *small thickness* compared with the aggregate thickness of the whole mass. Writing $\Sigma\frac{ph}{k}$ and $\Sigma\frac{p'h'}{k'}$ at full length, we obtain

$$\frac{-k_r\frac{d\zeta}{dz}}{-k'_{r'}\frac{d\zeta'}{dz'}}=\frac{\frac{h'}{k'}+\frac{h'_1}{k'_1}+\ldots+\frac{h'_{r'}}{k'_{r'}}+\ldots}{\frac{h}{k}+\frac{h_1}{k_1}+\ldots+\frac{h_r}{k_r}+\ldots}; \qquad (5.)$$

and if h_r and $h'_{r'}$ be each large, according to our first hypothesis, we shall have

$$\frac{-k_r\frac{d\zeta}{dz}}{-k'_{r'}\frac{d\zeta'}{dz'}}=\frac{\frac{h'_{r'}}{k'_{r'}}}{\frac{h_r}{k_r}}=\frac{k_r}{k'_{r'}}\cdot\frac{h'_{r'}}{h_r}; \qquad (6.)$$

and

$$\frac{\frac{d\zeta}{dx}}{\frac{d\zeta'}{dz'}}=\frac{h'_{r'}}{h_r}. \qquad (7.)$$

The first of these equations shows that, in the case before us, the quantity of heat transmitted will be in a ratio compounded of the ratios of the conductive powers, and the inverse ratio of the thicknesses; and the second equation shows that the increase of temperature in descending, or its decrease in ascending, will be inversely as the thickness of the strata, and independent of the conductive powers.

In the second case, the numerator and denominator of the fraction on the right-hand side of equation (5.) may be considered independent of k_r and $k'_{r'}$, on account of the comparative smallness of h_r and $h'_{r'}$. Hence, writing for the numerator $\frac{H'}{K}$ and for the denominator $\frac{H}{K}$,

$$\frac{-k_r\frac{d\zeta}{dz}}{-k'_{r'}\frac{d\zeta'}{dz'}}=\frac{K}{K'}\cdot\frac{H'}{H}, \qquad (8.)$$

and

$$\frac{\frac{d\zeta}{dz}}{\frac{d\zeta'}{dz'}}=\frac{K}{K'}\cdot\frac{H'}{H}\cdot\frac{k'_{r'}}{k_r}. \qquad (9.)$$

Hence, in this case, the quantities of heat transmitted will be independent of the conductive powers; while the rate of increase of temperature descending through any two strata situated respectively in the two groups of strata, will be in the inverse ratio of the conductive powers.

16. Let us now examine how far this investigation may apply to the case of the earth, assuming the actual terrestrial temperature to be due to the remains of a primitive heat. If the earth, considered spherical, were formed of matter of uniform conductive power, and the mean temperature at every point of its surface were the same, the surfaces of equal temperature would, after a sufficient lapse of time, be spherical surfaces concentric with the external surface; and in the actual case, making due allowance for the differ-

ences of mean superficial temperatures in different latitudes, and for possible variations in the conductive powers at different points, it would still seem extremely improbable that the above law respecting the surfaces of equal temperature should not be approximately true at depths beneath the surface at which such superficial causes as are merely local cannot be supposed to exercise a very sensible influence. Let us suppose then that there is a surface of equal temperature concentric with the earth's surface at the depth, for example, of fifty miles; and let us also take a portion of the surface of 100 miles in diameter, with one set of sedimentary strata, the thickness of which is small compared with the above depth; and another smaller area with another similar set of strata. If these portions of the earth's crust be contiguous, the flow of heat near their common boundary will not be entirely *vertical* (as supposed in our investigations), but in some degree *lateral* also; because the conductive powers being by hypothesis different, the temperatures at the same depth in the two sets of strata respectively will not be exactly the same, except at the lowest surface of each set, where they are here assumed to be the same. This difference of temperatures, however, at equal depths will be too small to produce any sensible lateral flow of heat except at points near the common junction of the two sets of strata, or therefore to affect sensibly the vertical flow of heat in all the more central parts of these two contiguous portions of the earth's superficial crust. Also, it is manifest that the surfaces of the strata having no more horizontal extent than here supposed, may be considered as plane instead of spherical. Hence, the hypothesis of the flow of heat being vertical will be very approximately true in the example now proposed.

Again, on account of the great pressure to which the portions of the earth's mass at considerable depths must be subjected, and the absence of stratification in the portion beneath the sedimentary deposits, it would seem impossible that the influence of discontinuity in the lower parts of the mass should not be considerably smaller than in the superincumbent stratified mass; and since it is small, as I have shown, in the latter, it must *à fortiori* be so in the former. It also follows from the great thickness of the whole mass here assumed, and the general values of $\frac{k}{p}$ determined in our own experiments $\left(\text{supposing them much the same as those of } \frac{k}{c}\right)$, that the value of $\Sigma\left(\frac{ph}{k}\right)$ must be very large. Hence $1+\Sigma\left(\frac{p}{q}\right)$ may be neglected in comparison of $\Sigma\left(\frac{ph}{k}\right)$; and for the same reason $1+\Sigma\left(\frac{p'}{q'}\right)$ may be neglected in comparison with $\Sigma\left(\frac{p'h'}{k'}\right)$. Also, if we suppose the two strata denoted by their conductive powers k_r and $k'_{r'}$, to be any two sedimentary strata, their thicknesses h_r and $h'_{r'}$ must be very small compared with the depth, which, for greater distinctness of conception, I have supposed to be fifty miles. Hence the equations (8, 9.) will be applicable to our present case. Moreover it seems extremely improbable that portions of the earth's mass beneath the sedimentary beds, and for considerable depths, should vary much in their conductive powers. All the igneous rocks I have tried are of high, but not very variable conductivity; and from all we can know by observation, we have no reason to suppose that the unstratified portion of the earth's crust presents any variation of general structure which would be likely to influence very materially its conductivity in different localities. For these reasons we may, I conceive, confidently assume that for any such depth as fifty miles, we shall have

$$\Sigma\left(\frac{h'}{k'}\right)=\Sigma\left(\frac{h}{k}\right),$$

or

$$\frac{H'}{K'}=\frac{H}{K},$$

very approximately. In this case equation (9.) becomes

$$\frac{\frac{d\zeta}{dz}}{\frac{d\zeta'}{dz'}}=\frac{k'_{r'}}{k_r};$$

i. e. the rates of change of temperature in ascending or descending will be very nearly in the inverse ratio of the conductive powers. This interpretation applies immediately to two strata in the two separate groups of strata; but it manifestly applies also to any two strata in the same group; since for each group the flow of heat is constant, and therefore for every stratum

$$k\frac{d\zeta}{dz}=\text{const.}$$

We also observe, that when

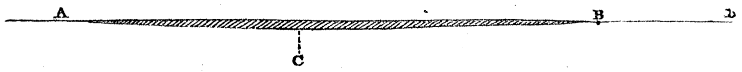
$$\frac{H'}{K'}=\frac{H}{K},$$

as above supposed, equation (8.) shows that the flow of heat is the same in both groups of strata.

17. The result at which we have arrived respecting the rate of change of temperature in ascending or descending through different strata, is of so much importance in testing the theory of terrestrial temperature which attributes the actual temperature of the earth's crust entirely to central heat, as we are now supposing, that it may be worth while to elucidate the above general conclusions by a particular and simple case.

Let us suppose, then, a single stratum AB (fig. 2) of sedimentary matter of about 100

Fig. 2.



miles in radius as above supposed, superimposed on the unstratified mass (C) of the earth's crust, which is supposed to remain uncovered with sedimentary beds in all the neighbouring region. Let us also suppose the thickness of our sedimentary bed to be about 1000 feet, and its conductive power to be one-third of that of the unstratified mass. Now let us first conceive the entire absence of the stratified mass. The temperature at the depth of 1000 feet, according to the actual observed rate of increase of temperature as we descend (about 1° FAHR. for every 60 feet), would be nearly

$$=T+16^{\circ};$$

and the temperature at the depth of fifty miles would be nearly

$$=T+4400°.$$

Let us now conceive the 1000 feet of sedimentary mass of lower conductive power to be deposited upon the unstratified mass. The escape of the heat would at first be impeded till the temperature of the bottom of the sedimentary mass should exceed the former temperature ($T+16°$) at the depth of 1000 feet, by a quantity which would compensate for the lower conductive power of the sedimentary portion. This would require the rate of increase of temperature in descending through this mass to be three times as great as in the unstratified mass, since the conductive power of the former is here assumed to be one-third of that of the latter. Thus, when the increase in descending 1000 feet should become 48° instead of 16°, the quantity of heat conducted through the upper stratum would be equal to that conducted to it through the lower mass, and the temperatures would again be stationary; but the rate of increase of temperature in the sedimentary mass would be three times as great as in the unstratified mass either beneath the sedimentary portion, or in the surrounding region beyond its boundaries. This is the result which our general formula applied to this case immediately affords.

In this explanation it has been assumed that the quantity of heat conducted through the sedimentary mass to the surface would be as great as that conducted to the surface immediately by the unstratified and more highly conductive mass. That this would be very approximately true is easily shown. The quantity of heat conducted through the lower and unstratified mass, of which we have assumed the thickness to be fifty miles, will depend on the difference of temperatures at its lower and upper surfaces. Now if the sedimentary mass did not exist, the temperature of the upper surface of the unstratified mass would $=T°$; and when it did exist, the temperature of the upper surface of the unstratified mass immediately below the sedimentary mass would $=T+48°$ when the temperature of the whole should have become *steady*, as above explained. Hence the quantity of heat conducted through the lower mass without the sedimentary bed, will be, to that conducted through it with the sedimentary bed, in the ratio of $4400°-T°$ to $4400°-(T°+48°)$, which is very nearly a ratio of equality.

18. This numerical example may also enable us to explain very simply the amount of lateral conduction. The temperature at the depth of 1000 feet at C would be nearly, as we have seen, $T°+48°$; while at the same depth beyond A and B, it would be $T°+16°$, the rate of increase there in descending being, by hypothesis, only one-third of that rate at C. Consequently the decrease of temperature in the horizontal distance CB, which we have supposed to be some 100 miles, would be about 32°; while the decrease for half the same distance (50 miles) along the vertical line through C, would be about 4300°. These two numbers enable us to judge how very small must be the lateral conduction of heat, in a case like that here considered, in comparison with the vertical conduction, and not only at the centre C of the area occupied by the stratum of smaller conductivity, but also throughout all the central portion of that area.

19. We are now prepared to compare our theoretical results with those obtained by observation respecting the terrestrial temperature at various depths beneath the earth's surface, as already described. Those observations, as before stated, extend to the greatest depths at which satisfactory observations on terrestrial temperature (so far as I am acquainted with them) have been made. We see that, according to our theoretical results, the rate of increase of temperature in different strata, either in the same or in different localities, ought to vary inversely as the conductive powers of the strata; whereas the observed rates of increase, in the instances above cited, exhibit a striking uniformity, although the observations are made in masses in which the conductive powers are unquestionably very different. I would direct attention especially to two of these cases, the Puits de Grenelle at Paris, and the coal-shaft at Duckenfield. In the former a great depth of chalk was penetrated, and in the latter a like depth of strata, which for the most part are hard arenaceous rocks. The chalk of the northern region of France and that of this country are extremely similar in composition and structure; we cannot therefore be much in error, I conceive, if we estimate its conductive power at from ·25 to ·27 (art. 10). Nor should I estimate the conductive powers of the superincumbent beds (of comparatively small thickness) at much higher values. At Duckenfield the mass which has been penetrated is more distinctly stratified, and consists of beds of a much greater variety than is found in a mass of chalk. They have, however, for the most part, the common character of being very siliceous; and many of the thickest beds, the aggregate of which composes a large portion of the mass through which the shaft passes, are almost entirely siliceous, and extremely hard and compact. I obtained the conductive powers of a considerable number of these rocks. They varied between ·51 and ·726. I estimate their mean value at about ·62, and the conductive power of the aggregate mass of all the strata at not less than ·5. The number of discontinuities between successive strata is not sufficient to diminish materially the conductivity of the whole; but whatever effect may be produced by them, it is probably more than counterbalanced by the effect of moisture in those strata which are less compact and of smaller conductive power *.

According to these estimates, then, the conductive power of the mass through which the Puits de Grenelle has penetrated, can scarcely exceed one half of that of the aggregate of the beds at Duckenfield. The rate of increase of temperature in the former case ought, therefore, according to our theory, to be nearly twice as great as in the latter, whereas it is 60 feet for 1° Fahr. at Paris, and only 65 feet at Duckenfield, instead of 110 or 120 feet, as the theory would have led us to suppose.

The instance above mentioned of the Artesian well at Mondorff (art. 12) presents a similar discrepancy between theory and observation, for there can be no doubt but that the mass of rocks through which that well passes, as above stated, is of considerably higher conductivity than the Chalk, though observation there gives a somewhat *more*

* The shaft at Duckenfield is likely, I believe, to attain its ultimate depth of upwards of 2000 feet in a short time. I hope to obtain from its lower portion much more complete evidence than I yet possess.

rapid increase of temperature than in the latter formation, the increase being in this case only 57 feet for 1°. The other cases referred to are more indefinite, on account of the omission of any detailed mention of the nature of the beds through which the heat is transmitted.

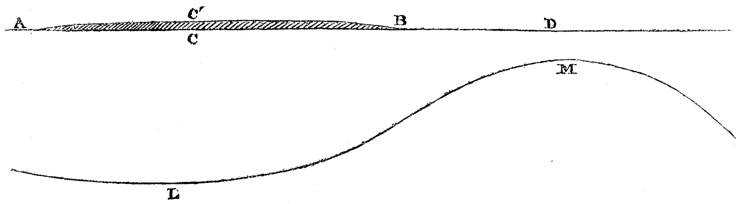
20. There are also other observations to which I have not yet alluded, the correctness of which in their general results may be depended on, though some of them may inevitably, perhaps, have been made under circumstances calculated to diminish their weight when considered separately. I allude to the observations made by Mr. HENWOOD, with great care and labour, and published as an Appendix (No. 1) to his work 'On the Metalliferous Deposits of Cornwall and Devon.' The heat of the mines of that district, situated in slate rock, is well known to be greater than that of the mines situated in granite. From a number of observations Mr. HENWOOD concluded that, on the average, and at depths not less than 600 feet, there was an increase of 1° FAHR. for 51 feet in the granite, and for 37·2 feet in the slate*. I have not been able to ascertain the conducting power of the Cornwall slate, and cannot therefore determine how far these results are consistent with each other, according to theory; but there can be no doubt but that the increase ought to be much more rapid, instead of being slower, in chalk than in either granite or slate; for all the older compact rocks, on which I have experimented, have been found to have comparatively high conducting powers. This increases still further the discrepancy between observation and the theory we are discussing.

21. In the preceding discussion it has been supposed that the isothermal surfaces at depths (such, for instance, as 50 or 60 miles) below the influence of local external causes, are approximately concentric with the surface of the earth itself, in which case it has been shown that the quantity of heat transmitted through the external shell must be very nearly the same in every part of it. It may, however, be conceived that these deeper isothermal surfaces may deviate from concentric spherical forms much more than has been here supposed. It would be difficult, I think, to assign any probable cause for such considerable deviations according to our theory; but admitting, for a moment, the hypothesis, let us examine what consequences will flow from it, and how far it will enable us to account for the general uniformity of the rate of increase of terrestrial temperature as we descend in different localities throughout a region like Western Europe, or for the local variations to which it is subject.

For this purpose let us take the particular example of Art. 17, except that we here suppose the deep isothermal surface LM to be of the form represented in fig. 3, instead of being parallel to the internal surface; and since the conductive power of the unstratified mass CLMD has been supposed three times as great as that of the sedimentary

* Since this paper was read, I have been informed by Mr. FOX, that, generally, those Cornish mines which present the most rapid increase of heat in descending have considerable quantities of water, a portion of which may probably ascend from lower depths, and increase the temperature of the higher portions of the mass. The rate of increase of temperature in the *dry* Cornish mines does not appear to differ much from the general rate of 1° FAHR. for about 60 feet of depth.

Fig. 3.



mass AC′BC, suppose the depth C′L (very nearly the same as CL) to be three times as great as DM. Also let t_1 denote the temperature at any point of LM, t_2 that at the point C, a central point at the lower surface of the sedimentary mass, and t_2' the temperature at D. The temperature of the surface at C′ will be very nearly that of the mean external temperature, and may therefore be considered equal to t_2', the superficial temperature at D. Consequently, in the particular example we are considering, we shall have

$$t_2 = t_2' + 48°,$$

and the difference of temperature, at L and C,

$$= 4400° - t_2,$$

$$= 4400° - (t_2' + 48°);$$

while the difference of temperatures at M and D will

$$= 4400 - t_2'.$$

Hence if Q and Q′ be the quantities of heat transmitted along LC and MD respectively in the same time,

$$Q : Q' :: \frac{4400° - (t_2' + 48)}{LC} : \frac{4400° - t_2'}{MD};$$

and since, by hypothesis, LC=3.MD, we shall have

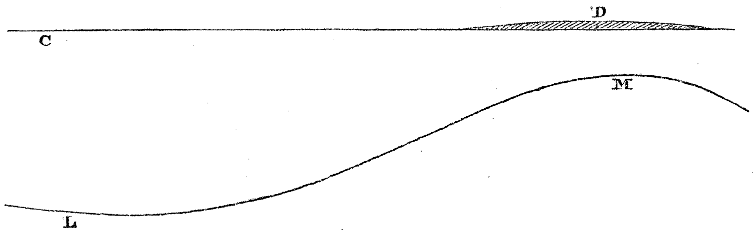
$$Q' = 3Q$$

very nearly. Consequently the quantity of heat which will be conducted through MD in a given time, will equal three times that conducted through CC′ in the same time; and since the conductive power in the former case is three times that in the latter, the rate of decrease of temperature through CC′ and through MD will be very nearly equal.

If, then, we could admit the hypothesis, that the depth of any deep internal surface of equal temperature beneath any point of the earth's external surface is inversely proportional to the conductive power of the superficial stratum at that point, we could thus account for the equality of the rates of increase of temperature in descending through superficial masses of different conductive powers. There does not appear, however, to be any conceivable grounds for the admissibility of this very limited hypothesis, according to the theory of central heat; for though irregularities in the structure and conductivity of the unstratified portion of the earth at such depths might produce considerable irregularities in the forms of the deep isothermal surfaces, it is difficult to imagine the possibility of any such relation as that above mentioned, between these irregularities of form and the existence of superficial sedimentary masses of lower conductivity. In fact, if the form of the isothermal surface LM (fig. 3) were established before the formation of the sedimentary mass AB, the tendency of that mass, when subsequently formed, would be to reduce LM more nearly to parallelism with the external surface; because the escape of heat from M being greater than that from L, the effect would be to lower the surface at M relatively to its position at L.

But if the hypothesis of the particular relation represented in fig. 3 between the depression of the isothermal surface at L, and the existence of a sedimentary mass like AB, directly above it, be deemed, as I conceive it must be, far too restricted to be admissible, we must consider our hypothesis respecting the non-concentric form of the deep isothermal surfaces independently of this additional restrictive hypothesis. Now if we do not admit this restriction, there is no reason why the sedimentary mass should not be formed above M instead of L (fig. 4). But in such case the quantity of

Fig. 4.



heat to be transmitted through it would be three times as great as at C, while the conductive power at the former point would only be one-third of that at the latter. Consequently the rate of increase of temperature in descending at D would be nine times as great as that at C. Hence the same hypothesis which might enable us to account for a rate of increase of terrestrial temperature, independent of conductive powers, in one region, would lead to the almost necessary conclusion that there must be found in the comparison of these rates of increase in different regions, deviations from the above law far greater than any which have been established by past observations, or are likely to be established by future ones.

I have here supposed (for the sake of speaking of a simple ratio) the conductive power of the unstratified mass to be three times that of the sedimentary mass deposited upon it. This ratio of three to one may be greater than that actually presented to us in the earth's crust. The actual ratio, however, is probably more than two to one, and abundantly sufficient, therefore, to justify the above conclusion.

22. There are other hypotheses, also, which would account for the equality of the rate of increase of temperature in descending beneath the surface, independently of the conductive power of the mass, in the simple case above supposed of a uniform sedimentary mass superimposed on the general mass of unstratified rock. It should be remarked, however, that in this case, as considered in the previous article, the equal rate of increase of temperature along the vertical lines through C′ and D respectively, would only hold

to the depth C′C. Beneath C the rate of increase would be only one-third of that above C, on account of the greater conductive power below than above that point. The like equality in the rate of increase in the case of art. 17, in which the isothermal surfaces are parallel to the outer surface, would result from the supposition that the portion of the unstratified mass along the vertical through D should have a conductive power three times as great as that of the unstratified mass along the vertical below C. This would seem absurd; and, besides, all particular and restricted hypotheses of this kind are liable to the objection mentioned in the preceding article. They do not enable us to account for the uniform rate of increase of temperature in different localities, unless we adopt also the hypothesis of certain necessary relations between the interior structure of the unstratified mass beneath, and the deposits of sedimentary beds on its surface—a supposition which must be deemed utterly inadmissible.

With respect to merely local deviations from the prevailing law throughout a region of considerable extent, we may observe that it is difficult to refer them to any deep-seated source of heat. It would seem much more probable that they are referable to some superficial action *.

On the whole then I cannot avoid the conclusion, that the existence of a central heat is not sufficient in itself to account for the phenomena which terrestrial temperatures present to us.

23. Let us now turn to the other case, in which, as expressed in equations (6, 7) (art. 15), the variation of temperature in ascending or descending is independent of the conductive power, and in which the quantity of heat transmitted through a given horizontal area is proportional to that power. This, it must be recollected, involves the supposition that the two strata whose conductive powers are k_r and k'_{r}, are of great thickness as compared with that of the other portion of the mass. If, however, instead of taking a single stratum of which the conductive powers are respectively k_r and $k'_{r'}$, we should take groups of strata having mean conductive powers equal to those quantities, the same equations will be applicable. Thus if we take, for example, groups of strata like those penetrated at the Puits de Grenelle, the coal-shaft at Duckenfield, and the other places enumerated above as giving very nearly the same mean variation of temperature along a vertical line, the equations will be applicable, assuming h'_r approximately equal to h_r; and it proves that, in such a case, there must be some cause acting to produce and maintain a surface of equal temperature at no great depth beneath that to which the increase of temperature may proceed according to the same law. At the same time, the flow of heat from this isothermal surface must be proportional to the conductive power of the superincumbent strata. I have endeavoured to show that this cannot be the case when all the heat is transmitted from the central portion of the earth, because, in this case, the quantity of heat transmitted must depend on conditions existing at comparatively great depths, where they can have little relation to the more superficial conditions. Not only, therefore, must there be some other cause generating heat to maintain this isothermal surface, but it must also be such as shall generate the

* Or to water rising from a lower level (see note, page 832).

greatest quantity of heat beneath those strata of which the conductive power is the greatest.

24. In this explanation of the conditions required in order that the variations of terrestrial temperature as we descend beneath the earth's surface may be nearly independent of the conductive powers of the masses through which the heat is transmitted, it is assumed that the temperature (t_1) is the same at the lower surfaces of the two groups of strata compared with each other. This, however, is not itself an essential condition. For when the flow of heat through any mass has become *steady*, it is exactly the same at every point along the line of transmission, and, therefore (in the case of the earth), we may consider any cause generating the heat necessary to maintain this steady flow, to act at any depth without thereby changing the temperature of the superincumbent mass. The essential condition is that the quantity generated, at whatever depth it may be produced, must be inversely as the conductive power of the mass through which it has to be propagated*. Thus, in the case of the earth, assuming terrestrial temperature to be due to the generation of heat in the superficial crust of the earth, then, if we should further assume (for the sake of illustration) that this generation of heat only takes place in the non-sedimentary unstratified portion of the crust, a smaller quantity of heat must be generated in those parts of the unstratified mass which are subjacent to sedimentary masses of the lower degrees of conductivity, than in those parts which extend up to the earth's external surface. Nor does this appear difficult to conceive, if, in addition to some such hypothesis as that just mentioned, we suppose the cause producing the heat to extend only to comparatively small depths, for, in such case, a larger portion of the mass capable of having heat generated within it, would be within the operation of the cause producing that effect, in that region in which the unstratified mass should extend up to the external surface.

25. After the preceding investigations, it appears to me extremely difficult, if not impossible, to avoid the conclusion that a part at least of the heat now existing in the superficial crust of our globe is due to superficial and not to central causes. It should be remarked, however, that the argument thus afforded is not directly against the theory of a *primitive* heat, but only against the manifestation of the remains of such heat as the sole cause of the existing terrestrial temperatures at depths beyond the direct influence of solar heat. The argument in favour of the earth's original fluidity (a state only conceivable as the effect of heat), founded on the spheroidal form of the earth, remains unaffected. Whatever cogency it may have been supposed to possess, it possesses still. At the same time, all those collateral arguments derived from the existing temperature of the earth's crust, or the climatal changes which we believe to have taken place on its

* It must be carefully recollected that this conclusion is restricted to those cases in which the generation of heat takes place entirely beneath the mass in which the increase of temperature is to be independent of the conductive power. Any superficial cause would probably generate heat within this mass as well as beneath it, so that its temperature would depend partly on heat thus generated within it, and partly on that transmitted through it from below, in which case the amount of heat generated in a given time need not bear the determinate relation to the conductive power stated in the text.

surface, are deprived, I conceive, of a great part of their weight. Moreover, admitting only a part of existing terrestrial heat to be due to superficial causes, the flow of heat from the earth's central portions must be less by that amount than if the whole flow were due to central heat. Consequently the rate of increase of temperature *due to the flow of central heat* must be proportionally diminished, and the depth at which we should arrive at the temperature of fusion, proportionally increased. The conclusion, therefore, that the earth's solid crust is so thin as many geologists have believed it to be, as well as those theories resting upon that conclusion—whether of volcanic action, or of elevation and depression of the earth's surface, at least in more recent geological times—must be in a great degree invalidated. But on these points I may have some further remarks to make in my next communication. At present it is not my object to carry these speculations beyond the point at which we are now arrived.

§ III. *Description of the Apparatus for determining Conductive Powers, and the mode of conducting the Experiments.*

26. I now proceed to an account of the experiments, the general results of which have been given, and to the description of the apparatus by which they were made.

Fig. 5.

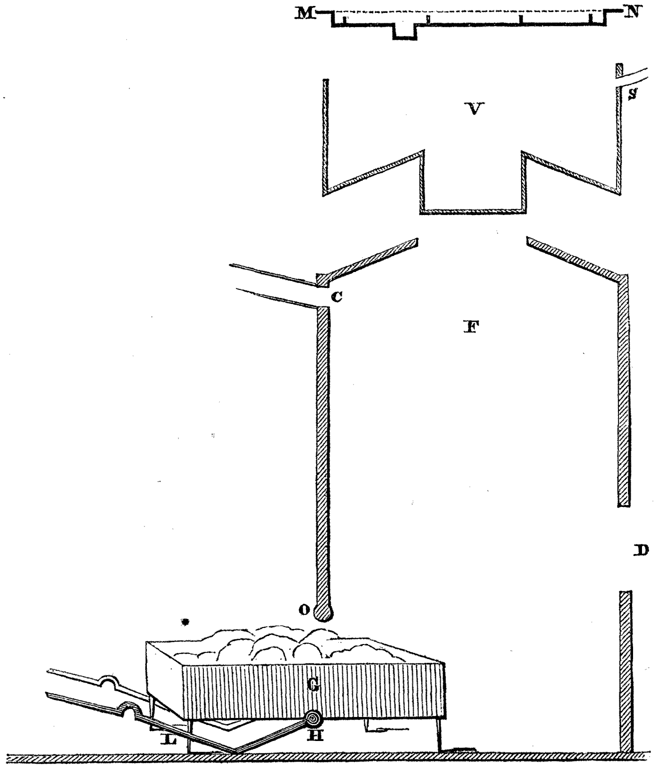


Fig. 6.

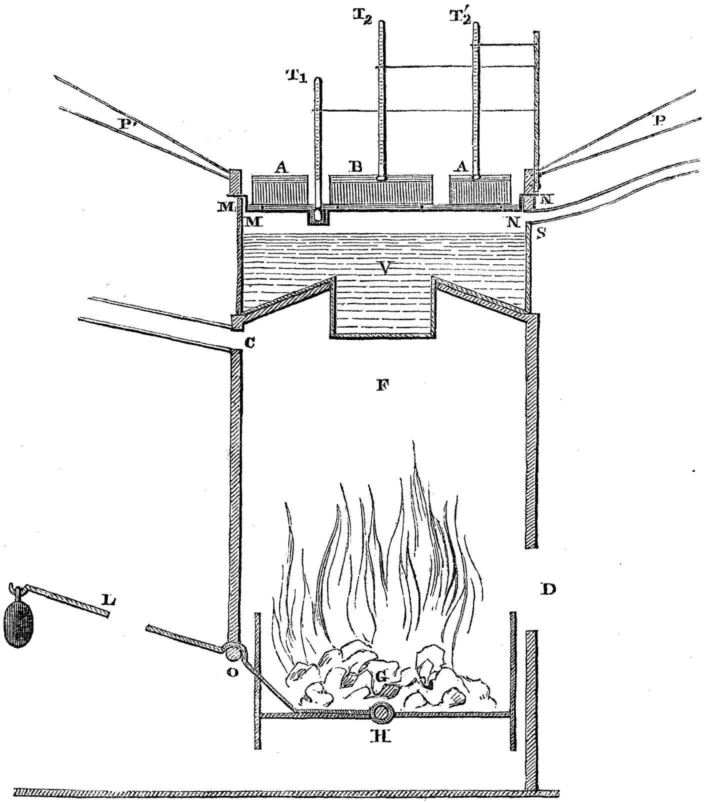


Fig. 5 represents the different parts of the apparatus separated from each other. In fig. 6 they are represented as fitted into each other preparatory to an experiment. F is the stove, the walls and roof of which are of cast iron. The horizontal section of it below the roof is rectangular; the orifice at the top is circular. C is the commencement of the chimney, which is carried horizontally and then vertically through the roof of the building, the horizontal portion being sufficiently long to prevent any sensible influence on the experiments arising from the heat radiating from the chimney. In the side of the stove is a rectangular opening D, which may be closed at pleasure, and through which fuel is supplied to the grate G. This grate fits the stove, but so as to be capable of moving freely up and down within it. The opening in the lower part of the stove at O is large enough to admit of the grate and the fuel contained in it being withdrawn entirely from the stove when required.

The horizontal section of the upper portion of the vessel V is rectangular; that of the part projecting downwards is circular, and such as to pass freely into the circular orifice at the top of the furnace, to the roof of which also this vessel is made to fit. MN is a shallow vessel, just fitting the top of V, on the upper rim of which it is supported by the projecting edges represented at M and N, when the parts are put together as in fig. 6. L there represents a lever working on a fulcrum at O, and so attached to the grate at the ends of an axis H, perpendicular to the plane of the paper, as to constitute a rough approximation to a parallel motion when the grate is moved vertically by heightening or lowering the opposite extremity of the lever, to which a counterbalancing weight is suspended. The lever, also, is easily removeable from the fulcrum O, and then becomes a handle by which the grate and its contents can be entirely withdrawn from the stove as above stated. Thus by raising or lowering, or by partially or entirely withdrawing the fire, the heat communicated to the vessel V can be regulated with considerable nicety as circumstances may require. A chimney, proceeding to a sufficient distance horizontally from the vessel V, carries off the steam when water is used in V at the boiling temperature.

Fig. 7.

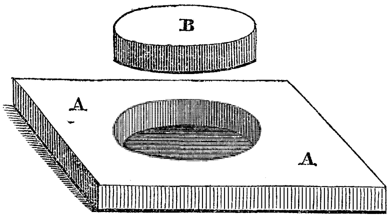


Fig. 7 represents one of the blocks, of which I had several of different thicknesses, and made of substances of different conductive powers. Its horizontal dimensions are such as to fit easily into the shallow vessel MN. In the centre of the block is a cylindrical orifice, five inches in diameter, for the reception of cylinders of any substance the conductive power of which is to be determined. In making an experiment, a sufficient quantity of mercury is poured into the shallow vessel MN, just to cover six points projecting from its bottom. The large block is then placed so as to rest on three of these points, and the inner cylindrical block to be experimented on is then inserted in the cylindrical orifice of the large block, and rests on the other three projecting points. The exact contact of the lower surfaces of both blocks with the mercury is thus secured. The outer block is also surrounded by a square frame of wood, without top or bottom, resting on the rim of the vessel V. The apertures between this frame and the sides of the large block, and that immediately surrounding the inner block, are filled with cotton-wool to prevent as much as possible the lateral transference of heat, both from the inner block to the outer one, and from the vertical sides of the latter into surrounding space. Also, from the upper edges of the outer block (or rather of the wooden frame surrounding it) two or three wooden floors with intervals between them, were constructed of loose boards, extending several feet on every side (sections of which are represented in fig. 6 at P and P′), and intended to prevent as far as possible the influence of radiation from the heated stove below, and the objects immediately surrounding it. The upper surfaces, both of the outer and inner blocks (fig. 7), are scooped out, leaving a projecting rim round their edges, so as to form shallow vessels for the reception of mercury; or the same object is attained by fixing round the edges a thin, narrow rim of iron. These spaces, in experimenting, are filled with mercury of just sufficient depth to cover the small spherical bulb of a thermometer. In fig. 6, A, B, A, represent the section of the outer and inner blocks, with the interspace between them. One thermometer (T_1) passes down this interspace into the mercury below the blocks; while T_2 gives the temperature of the upper mercury on the block B, the conductive power of which is to be determined; and T'_2 gives that of the mercury which covers the surface of the outer block. This latter mercury has sometimes been dispensed with, in which case the temperature of the uncovered surface of the outer block has been determined approximately by placing a thermometer in a small hole formed in the surface, and containing mercury.

This arrangement being completed, the fire is placed in the stove, and the lower mercury is heated to the required temperature, as indicated by T_1. The temperature much the most frequently used has been that of boiling-water, in which case the vessel V is kept nearly full, the exhaustion caused by the continual generation of steam being compensated by a supply through a tube arranged for the purpose. This temperature is preferable on account of the facility with which it can be preserved for any length of time. For higher temperatures, which were rarely used, and for which water cannot be employed, the vessel V is left empty. In all cases when the lower mercury has attained its required temperature, it must be kept as steadily as possible at that temperature, till the thermometer, T_2, which gives the temperature of the upper mercury on B, becomes stationary. The temperatures indicated by T_1 and T_2 are those denoted by t_1 and t_2 in the formula (5), (art. 1). The remaining temperature in that formula (τ) is obtained by waving a thermometer above and within a short distance of the blocks, the instrument being carefully guarded against the influence of direct radiation from surrounding objects.

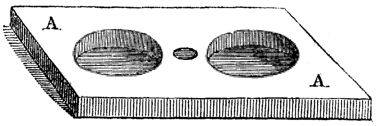
27. To secure entire accuracy by this mode of experimenting, it is manifest that the horizontal extent of the mass experimented on ought to be so large, compared with its thickness, that no appreciable diminution of the temperature (t_2) of the central portion of the upper surface should be produced by the *lateral* transference of the transmitted heat, the whole of which is supposed in our mathematical formula to be transmitted

vertically. It would have been almost impossible, however, to obtain slabs of any considerable extent from many of the harder and more intractable rocks on which I have experimented. But the fact is, that no great accuracy is required in experiments of this nature; for if the conductive power of any piece of rock were determined with the ultimate degree of accuracy, it would only tell us *approximately*, and not *accurately*, the conductive power of another piece of rock of the same kind, however close might be their resemblance in mineral structure; so that it must be always necessary, when the conductive power of any proposed substance is required with great accuracy, to determine that power by direct experiments on the substance itself. I have endeavoured, however, to take every precaution necessary to secure all the accuracy required with reference to my more immediate objects in these researches.

In order that the error resulting from the observations of temperatures may be as small as possible, it is manifestly desirable that $t_2 - \tau$ and $t_1 - t_2$ should both be as large as possible, and not very different in magnitude. But this condition, if the conductive power and the temperatures used were sufficiently great, would require a thickness of the block which might be undesirable in consequence of its limited extent of horizontal surface. The thickness which I have more generally used for mineral substances, is about 2 inches, and about 1 inch for substances, such as wax and spermaceti, of lower conductive powers, while the outer block has been about 15 inches square, and of very nearly the same thickness as the blocks experimented on, and covered with mercury in the same manner. Having several of these outer blocks made of substances of different conductive powers, I have always been able to select one for each experiment in which the conductivity was nearly the same as that of any proposed substance, taking care, however, that it should never be *greater*; for in such case the temperature of the surface of the outer block, as indicated by T'_2, fig. 6, would be higher than the proper temperature of the surface of B, and would, therefore, tend to raise also the observed temperature of B, as given by T_2, above the proper temperature, and thus to give the conductive power of B too great. I have been anxious that, on the contrary, the error should always be in *defect*, and therefore have always selected such a block for each experiment that the temperature given by T'_2 should generally be somewhat less than that given by T_2. A small error has, doubtless, been thus superinduced, especially in reference to substances of the highest conductive powers, but I am satisfied that it has in no case been of any importance as regards the conclusions which I have drawn from these results.

28. In all cases in which it has been necessary to compare the conductive powers of two blocks with as much accuracy as possible, I have adopted a somewhat different method from that above described. Instead of a single block (B) placed in the central cylindrical hole of the outer block (A), the two blocks to be compared have been placed in two cylindrical holes pierced through the outer block, as represented in fig. 8, and such that the two blocks

Fig. 8.



(B; B′) placed in them should be as nearly as possible under the same conditions during the experiment. The small orifice between them admits the thermometer T_1 for the determination of the temperature t_1, while t_2 is determined for each block separately. When this has been done, the two blocks are interchanged, each being thus placed in the hole previously occupied by the other, and the experiment repeated. The mean results of the two experiments have been adopted. The discrepancies, though small, are sufficient to show the necessity of the precaution where considerable accuracy is required.

29. It would be useless, I conceive, to record the details of all the numerous experiments made for the purpose of simply determining the conductive powers of particular substances. They were made in the manner already described (art. 25), and when the apparatus was once arranged, required little but patience to wait for the final indications of the thermometers. Such at least was the case when the steam from boiling water was used to produce the temperature of the lower surface of the block; when any other temperature was employed, constant attention was necessary to preserve it as steady as possible.

It may suffice to give the details of one experiment of this kind made with high temperatures, from which it may be understood how far the steadiness of the temperature t_1 could be maintained. In almost all cases, however, the same temperature, that of boiling water, was used for t_1 in the experiments now spoken of, in order that they might all be made under nearly the same conditions with respect to temperature. The experiments made to compare the conductive powers of two blocks, for the purpose of ascertaining the influence of pressure, moisture, discontinuity, &c., require greater care; and I have therefore thought it right to give all the principal ones of this kind in sufficient detail to enable the reader to judge of the evidence which they afford. The symbols used have the same signification as heretofore (art. 1, &c.). I repeat them here for the convenience of more immediate reference.

t_1= temperature of the mercury in contact with the lower surface of the cylindrical block B, on which the experiment is made;
t_2= that of the mercury on its upper surface;
t'_2= that of the upper surface of the surrounding or outer block A;
τ = that of the air immediately above the blocks A and B;
h= the length of the cylindrical block B;
k= conductive power of B;
c = radiating power of mercury.

It will be observed that in some of the following experiments the recorded value of t'_2 is greater than that of t_2, in opposition to the remark in the last paragraph of article 26. In such cases the temperature t'_2 really corresponded to points at somewhat a lower level in the outer block A than the upper surface of B to which t_2 corresponds. I only observed t'_2 to assure myself of there being no amount of *lateral* transference of heat which could materially affect the results of the experiments. The values of $\frac{k}{c}$ are found by substitution in the formula

$$\frac{k}{c}=\frac{t_2-\tau}{t_1-t_2}h.$$

One foot is adopted as the linear unit.

The blocks were cylindrical, the diameter of their bases being about 3 inches.

The blocks of spermaceti and wax, which were solidified under compression, were formed by pouring the substances in a fluid state into a cylindrical hole in a strong block of iron, and leaving them to cool and solidify under the required pressure, applied by means of a piston exactly fitting the cylindrical hole, and rendered by proper *packing* perfectly fluid-tight. The piston was acted on by a powerful lever. When these substances were compressed after solidification, cylindrical blocks of them were formed of the proper dimensions before the compressing force was applied to them. Blocks of chalk, clay, &c. were compressed in the same manner.

30. *Experiments made for the purpose of determining the effect of pressure on the conductivity of various substances.*

Spermaceti.

I. (*a*.) Spermaceti, solidified under a pressure p, on the square inch.

The two cylindrical blocks are denoted by I. and II.

	I. $\begin{cases} h=\cdot086 \text{ ft.} \\ p=900 \text{ lbs.} \end{cases}$		II. $\begin{cases} h=\cdot086 \text{ ft.} \\ p=6500 \text{ lbs.} \end{cases}$		
t_1. $91^\circ\cdot3$	t_2. $68^\circ\cdot2$	t'_2. 67°	t_2. $66^\circ\cdot6$	t'_2. 67°	τ. 45°
	$\frac{k}{c}=\cdot086$		$\frac{k}{c}=\cdot075$		

(*b*.) Blocks interchanged.

	I.	II.	
t_1. $90^\circ\cdot5$	t_2. $69^\circ\cdot3$	t_2. $68^\circ\cdot6$	τ. 47°
$\frac{k}{c}=\cdot09$		$\frac{k}{c}=\cdot086$	

$$\text{Mean values} \begin{cases} \text{I. } \frac{k}{c}=\cdot088 \\ \text{II. } \frac{k}{c}=\cdot08. \end{cases}$$

II. Two blocks of spermaceti, one (I.) solidified under pressure, the other (II.) compressed after solidification, the compressing force being in each case =7500 lbs. per square inch.

	I. $\left\{\begin{array}{l} h=\cdot086 \text{ ft.} \\ p=7500 \text{ lbs.}\end{array}\right.$		II. $\left\{\begin{array}{l} h=\cdot086 \text{ ft.} \\ p=7500 \text{ lbs.}\end{array}\right.$		
t_1. 94°·2	t_2. 70°·2	t'_2.	t_2. 70°·7	t'_2.	τ. 47°·2
	$\frac{k}{c}=\cdot089$		$\frac{k}{c}=\cdot086$		

Wax.

III. Block I. uncompressed, and II. compressed.

	I. $\left\{\begin{array}{l} h=\cdot09 \text{ ft.} \\ p=\text{atmosph.}\end{array}\right.$		II. $\left\{\begin{array}{l} h=\cdot09 \text{ ft.} \\ p=7500 \text{ lbs.}\end{array}\right.$		
t_1. 132°·5	t_2. 92°·5	t'_2. 95°·5	t_2. 94°·6	t'_2. 94°	τ. 61°
	$\frac{k}{c}=\cdot072$			$\frac{k}{c}=\cdot079$	

Chalk.

IV. Block I. uncompressed; II. compressed.

	I. $\left\{\begin{array}{l} h=\cdot13 \text{ ft.} \\ p=\text{atmosph.}\end{array}\right.$		II. $\left\{\begin{array}{l} h=\cdot11 \text{ ft.} \\ p=7500 \text{ lbs.}\end{array}\right.$		
t_1. 212°	t_2. 147°·3	t'_2. 154°·5	t_2. 148°·5	t'_2. 153°·5	τ. 55°
	$\frac{k}{c}=\cdot18$		$\frac{k}{c}=\cdot16$		

The fact of $\frac{k}{c}$ being smaller in the compressed than in the uncompressed block, may probably be attributed to some difference in the quantity of moisture contained in them.

V. (*a.*) Two compressed blocks.

	I. $\left\{\begin{array}{l} h=\cdot11 \text{ ft.} \\ p=4300 \text{ lbs.}\end{array}\right.$		II. $\left\{\begin{array}{l} h=\cdot11 \text{ ft.} \\ p=7500 \text{ lbs.}\end{array}\right.$		
t_1. 212°	t_2. 150°	t'_2. 159°	t_2. 152°	t'_2. 159°	τ. 61°
	$\frac{k}{c}=\cdot157$		$\frac{k}{c}=\cdot165$		

(b.) Last experiment repeated, with the blocks interchanged.

	I. $\left\{\begin{array}{l}h=\cdot 11 \text{ ft.}\\ p=4300 \text{ lbs.}\end{array}\right.$		II. $\left\{\begin{array}{l}h=\cdot 11 \text{ ft.}\\ p=7500 \text{ lbs.}\end{array}\right.$		
t_1. 212°	t_2. 152°	t'_2. 159°	t_2. 152°·4	t'_2. 159°	τ. 60°
	$\frac{k}{c}=\cdot 168$		$\frac{k}{c}=\cdot 17$		

Taking the mean of the experiments (a.) and (b.), we have for

$$\text{I., } \frac{k}{c}=\cdot 162\text{ ; and II., } \frac{k}{c}=\cdot 167.$$

Clay.

VI. (a.) Block I. uncompressed; II. compressed.

	I. $\left\{\begin{array}{l}h=\cdot 12 \text{ ft.}\\ p=\text{atmosph.}\end{array}\right.$		II. $\left\{\begin{array}{l}h=\cdot 12 \text{ ft.}\\ p=7500 \text{ lbs.}\end{array}\right.$		
t_1. 212°	t_2. 163°·1	t'_2. 168°	t_2. 171°·3	t'_2. 168°	τ. 52°
	$\frac{k}{c}=\cdot 27$		$\frac{k}{c}=\cdot 38$		

(b.) Blocks interchanged.

	I.		II.		
t_1. 212°	t_2. 162°	t'_2. 168°·5	t_2. 170°	t'_2. 170°	τ. 53°
	$\frac{k}{c}=\cdot 25$		$\frac{k}{c}=\cdot 33$		

The means of (a.) and (b.) give for I. $\frac{k}{c}=\cdot 26$; and for II. $\frac{k}{c}=\cdot 355$.

VII. (a.) Two compressed blocks of clay.

	I. $\left\{\begin{array}{l}h=\cdot 12 \text{ ft.}\\ p=4300 \text{ lbs.}\end{array}\right.$		II. $\left\{\begin{array}{l}h=\cdot 12 \text{ ft.}\\ p=7500 \text{ lbs.}\end{array}\right.$		
t_1. 212°	t_2. 168°·6	t'_2. 169°·2	t_2. 171°·6	t'_2. 170°·3	τ. 56°
	$\frac{k}{c}=\cdot 30$		$\frac{k}{c}=\cdot 34$		

(*b.*) Blocks interchanged.

	I.		II.		
t_1. 212°	t_2. 165°·6	t'_2.	t_2. 170°·5	t'_2.	τ. 56°
	$\frac{k}{c}=\cdot30$		$\frac{k}{c}=\cdot33$		

Mixed sand and clay in equal quantities.

VIII. (*a.*) Two compressed blocks.

	I. $\begin{cases} h=\cdot12 \text{ ft.} \\ p=4300 \text{ lbs.} \end{cases}$		II. $\begin{cases} h=\cdot12 \text{ ft.} \\ p=7500 \text{ lbs.} \end{cases}$		
t_1. 212°	t_2. 174°·3	t'_2. 167°·5	t_2. 175°	t'_2. 167°·5	τ. 58°·5
	$\frac{k}{c}=\cdot368$		$\frac{k}{c}=\cdot378$		

(*b.*) Blocks interchanged.

	I.		II.		
t_1. 212°	t_2. 173°	t'_2. 166°·5	t_2. 175°	t'_2. 166°5	τ. 58°
	$\frac{k}{c}=\cdot36$		$\frac{k}{c}=\cdot379$		

The means from (*a.*) and (*b.*) give for I., $\frac{k}{c}=\cdot364$; and for II., $\frac{k}{c}=\cdot378$.

31. *Experiments to determine the effect of Discontinuity.*

Two cylindrical blocks of sandstone (freestone) were formed from the same larger block. The experiment IX. was made to test the equality of their conductive powers.

IX. (*a.*) Two blocks of sandstone.

	I. $h=\cdot166$ ft.		II. $h=\cdot166$ ft.		
t_1. 211°	t_2. 174°·7	t'_2. 173°·6	t_2. 175°	t'_2. 175°·8	τ. 63°
	$\frac{k}{c}=\cdot51$		$\frac{k}{c}=\cdot51$		

(*b*.) Blocks interchanged.

	I.		II.		
t_1. 210°·9	t_2. 175°·5	t'_2. 175°	t_2. 176°	t'_2. 176°·8	τ. 65°
	$\frac{k}{c}$=·52		$\frac{k}{c}$=·53		

The means of (*a*.) and (*b*.) give for I., $\frac{k}{c}$=·515; and for II., $\frac{k}{c}$=·52.

X. One block was broken transversely, and the two parts again fitted accurately together without any interposing substance.

	Broken Block.		Whole Block.		
t_1. 210°·9	t_2. 166°·3	t'_2. 175°·2	t_2. 176°	t'_2. 175°·5	τ. 65°

XI. The two parts of the broken block cemented together by plaster of Paris applied wet and allowed to dry and harden. The length of the block was thus increased one-twentieth of an inch.

	Broken Block.		Whole Block.		
t_1. 210°·2	t_2. 170°	t'_2. 172°·6	t_2. 174°	t'_2. 172°	τ. 57°

XII. Finely powdered dry clay placed between the two parts of the broken block. The length was increased by one-tenth of an inch.

	Broken Block.		Whole Block.		
t_1. 210°·2	t_2. 165°·6	t'_2. 173°	t_2. 174°	t'_2. 173°·2	τ. 59°

XIII. Without removing the blocks the temperature was reduced.

	Broken Block.		Whole Block.		
t_1. 154°·6	t_2. 126°·3	t'_2. 131°·3	t_2. 131°	t'_2. 129°·2	τ. 56°

XIV. (*a*.) The two portions of the broken block cemented by moist clay, and pressed together with a pressure of 140 lbs. per square inch for twelve hours. The length was =·18 foot.

	Broken Block.		Whole Block.		
t_1.	t_2.	t'_2.	t_2.	t'_2.	τ.
211°	168°·2	173°·4	173°·4	171°·7	56°

(*b.*) Blocks interchanged.

	Broken Block.		Whole Block.		
t_1.	t_2.	t'_2.	t_2.	t'_2.	τ.
211°	167°·9	172°	175°	175°·5	59°

XV. The last experiment repeated at a lower temperature.

	Broken Block.		Whole Block.		
t_1.	t_2.	t'_2.	t_2.	t'_2.	τ.
149°·8	124°·3	125°·7	128°·6	128°	57°

32. *Experiments to ascertain the influence of Moisture on the conductive powers of Rocks.*

XVI. (*a.*) A block of chalk (from the Lower Chalk near Cambridge), very moist.

l=·16 feet.

t_1.	t_2.	τ.
211°·5	162°	65°

$$\frac{k}{c}=\cdot 3.$$

(*b.*) The same block saturated with moisture, tried at a lower temperature.

t_1.	t_2.	τ.
123°·5	101°·4	60°

$$\frac{k}{c}=\cdot 3.$$

(*c.*) The same block well dried.

t_1.	t_2.	τ.
211°·5	147°	70°

$$\frac{k}{c}=\cdot 19.$$

Another similar block gave almost identical results.

XVII. (*a.*) Sandstone block, dry.

$l = \cdot 17$ feet.

t_1.	t_2.	τ.
211°	173°	63°

$\frac{k}{c} = \cdot 49.$

(*b.*) Same block, saturated with moisture.

t_1.	t_2.	τ.
129°	114°·4	60°

$\frac{k}{c} = \cdot 62.$

XVIII. (*a.*) New Red Sandstone block, dry.

$l = \cdot 16$ feet.

t_1.	t_2.	τ.
126°·5	100°·8	59°

$\frac{k}{c} = \cdot 25.$

(*b.*) Same block, saturated.

t_1.	t_2.	τ.
210°·4	179°·5	61°·5

$\frac{k}{c} = \cdot 6.$

XIX. (*a.*) Millstone Grit, very hard and compact, dry.

$l = \cdot 166$ feet.

t_1.	t_2.	τ.
210°·25	182°	60°

$\frac{k}{c} = \cdot 71.$

(*b.*) Same block, saturated with moisture.

t_1.	t_2.	τ.
210°·5	181°·2	59°

$\frac{k}{c} = \cdot 69.$

33. Other similar experiments were made, of which it seems unnecessary to repeat the details. All the results obtained have been already discussed (art. 10). I shall select an example of the experiments for simply determining the conductive power of any proposed substance. In such cases a single block (B) was generally used with an outer block (A) having a single cylindrical hole. The base of the block experimented on (B) was usually between 4 and 5 inches in diameter. In this example the temperature t_1 is much higher than that of boiling water. The reader will be able to judge from it the degree of approximation to stationary temperatures which I was able to obtain, when not availing myself of the constant temperature of boiling water. For low temperatures there was no difficulty in maintaining the temperatures very nearly stationary.

XX. Block of Sandstone.

$h = \cdot 17$ feet.

Time.	t_1.	t_2.	t_2'.	τ.
h m				
3 30	437°·0	......	......	79°
3 45	448·0			
3 50	447·5			
3 55	447·5	314°·8		
4 0	447·5			
4 5	448·2			
4 10	448·2	320·8		
4 15	447·8			
4 20	447·6			
4 25	447·1	323·8		
4 30	447·8	324·6		
4 35	447·7	325·3		
4 40	447·6	326·0		
4 45	448·1	326·4		
4 50	447·0	326·3	......	81·5
4 55	446·8			
5 0	447·8	326·8	330°·5	
5 5	447·4			
5 10	448·0	327·3	......	81·0
5 30	......	{ Continued slow oscillations through 1°·5. }	......	81·0

Taking the mean of all the values of t_1, from $3^h\ 45^m$ to $5^h\ 10^m$ inclusive, we have $t_1 = 447°\cdot65$; and taking the mean of the values of t_2, from $4^h\ 40^m$ to $5^h\ 10^m$ inclusive, we have $t_2 = 326°\cdot5$ nearly. Also taking $\tau = 81°$, we obtain

$$\frac{k}{c} = \frac{245\cdot5}{121\cdot15}(\cdot17) = \cdot34.$$